Master Thesis

Strong games played on random graphs

Did you ever play games such as Tic-Tac-Toe, its close relative n-in-a-row or Hex? Then you are already familiar with strong games. Strong games, as a specific type of Positional games, involve two players alternately claiming unoccupied elements of a set $X$, which is referred to as the board of the game. The two players are called Red (the first player) and Blue (the second player). The focus of both players is a given family $\mathcal{H} \subseteq 2^X$ of subsets of $X$, called the winning sets of the game. While playing, Red and Blue take turns in claiming previously unclaimed elements of $X$, exactly one element in each round, with Red starting the game. The winner of such a game $(X, \mathcal{H})$ is the first player to claim all elements of some winning set $F \in \mathcal{H}$. If this has not happened until the end of the game, i.e. until all elements of $X$ have been claimed by either Red or Blue, the game is declared as a draw.

In a little more abstract sense, Positional games can also be played on the edge set of a graph $G = (V, E)$. In this case, $X = E$ and the winning sets are all the edge sets of subgraphs of $G$ which possess some given graph property $P$, such as “being connected”, “containing a perfect matching”, “admitting a Hamilton cycle”, “being not $k$-colorable”, “containing an isomorphic copy of given graph $H$” etc.

In this thesis, we would be considering strong games played on the edge set of a $n$-vertex random graph $G \sim G(n, p)$, where each edge of the complete graph $K_n$ is kept with probability $p$, independently at random. Recently, the study of strong games played on the edge set of a typical random graph was initiated in [1]. In particular, the perfect matching game played on $G \sim G(n, p)$, where $0 < p \leq 1$ is a constant, was analyzed and Red was provided with a winning strategy. The aim of this thesis would be to further analyze strong games played on the edge set of a random graph. Interesting approaches would be to find winning strategies for Red in other games (e.g. the Hamilton cycle game), or to analyze the perfect matching game for smaller, non-constant edge probabilities $p$.

Prerequisites: Basic knowledge of graph theory and discrete probability.

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