The Erdős-Rényi random graph process $G_0, G_1, G_2, \ldots$ is defined as follows: $G_0$ is the empty graph on $n$ vertices and $G_{t+1}$ is obtained from $G_t$ by adding an edge between two randomly chosen vertices in $G_t$.

An Achlioptas process is a generalisation of this process which contains an additional deterministic component. In such a process, $G_{t+1}$ is obtained from $G_t$ by choosing two pairs of vertices and then adding one of those pairs to $G_t$, according to some rule. As an example, the rule might say that one should pick the first pair of vertices if and only if both vertices are isolated vertices in $G_t$. Note that the Erdős-Rényi process corresponds for example to the rule that always picks the first edge.

The goal of this thesis is to study the effect of the rule on the $k$-connectivity transition of the process. A graph $G$ is said to be $k$-connected if for every subset $X$ of at most $k-1$ vertices, the graph $G - X$ obtained by removing the vertices from $X$ is still connected. Note that for $k = 1$, $k$-connectivity is the same as connectivity.

For the Erdős-Rényi process, it is known that for every $k \in \mathbb{N}$ and any function $\omega(n)$ such that $\lim_{n \to \infty} \omega(n)/n = \infty$, we have

$$\lim_{n \to \infty} \Pr[G_t \text{ is } k\text{-connected}] = \begin{cases} 0 & \text{if } t = \frac{n \log n + (k-1)n \log \log n - \omega(n)}{2} \\ 1 & \text{if } t = \frac{n \log n + (k-1)n \log \log n + \omega(n)}{2} \end{cases}.$$ 

Thus, the process becomes $k$-connected at a time around $\frac{n \log n + (k-1)n \log \log n}{2}$. This function is called the threshold function for $k$-connectivity. The goal of the thesis would be to determine the threshold function for $k$-connectivity for more general Achlioptas processes.

For $k = 1$ (i.e., for connectivity), the threshold is known quite precisely for a large class of rules, called bounded size rules [1]. These are rules that base their decision only on the component sizes of the endpoints of the candidate edges, treating all component sizes larger than some constant $K$ in the same way. For example, the rule given as an example above is a bounded size rule.

It turns out that there are bounded size rules which become connected at a very different time than the Erdős-Rényi rule. The goal of the thesis would be to see if this is still true for larger $k \geq 2$. If the behaviour of bounded size rules turns out not to be very interesting, then one could look at a slightly larger class of rules.

**Prerequisites** Basic knowledge in graph theory and discrete probability, and some knowledge of randomized algorithms. If you attend/have attended “Randomized Algorithms”, you are perfectly prepared for this topic.

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