1. Consider the following famous theorem of Dirac (1952):

**Theorem 1** Let $G$ be a graph on $n \geq 3$ vertices. If the minimum degree of $G$ is at least $n/2$, then $G$ is Hamiltonian.

(a) Prove Dirac’s Theorem.

(b) Use Dirac’s Theorem to prove that for an arbitrarily small $\varepsilon > 0$, $G(n, 1/2 + \varepsilon)$ is a.s. Hamiltonian.

**Hint:** use Chernoff’s bound, stating that if $X \sim \text{Bin}(n, p)$, then $\Pr(X \leq (1 - \varepsilon)np) \leq e^{-\varepsilon^2 np/2}$, where $0 \leq \varepsilon < 1$.

2. Let $0 < \varepsilon \leq 1/2$ be arbitrarily small, let $p = n^{\varepsilon-1/2}$ and let $G = G(n, p)$.

(a) Prove that $G$ is a.s. $\sqrt{n}$-vertex-connected (a graph $G$ is called $k$-vertex-connected if $G[V(G) \setminus S]$ is connected for every $S \subseteq V(G)$ such that $|S| < k$).

(b) Prove that a.s. $G$ contains a Hamilton cycle.

**Hint:** use the following theorem ($\kappa(G)$ is the vertex-connectivity of $G$, that is $\kappa(G) = k$ if $G$ is $k$-vertex-connected but it is not $(k + 1)$-vertex-connected):

**Theorem 2 (Chvátal and Erdős 1972)** If $|V(G)| \geq 3$ and $\kappa(G) \geq \alpha(G)$, then $G$ is Hamiltonian.