1. Let \( G = G(n, p) = (V, E) \), where \( n \) is sufficiently large, and \( p = \frac{1}{n} (\log n + \log \log n + o(1)) \). Let \( S \) denote the set of vertices of \( G \) which have degree at most \( \frac{\log n}{\log \log n} \).

(a) Prove that a.s. \( |S| = o(\sqrt{n}) \).

(b) Prove that a.s. \( S \) is an independent set.

**Hint:** Use the following corollary of the FKG inequality: Let \( P_1 \) be a monotone increasing graph property and let \( P_2 \) be a monotone decreasing graph property; then \( \Pr(G(n, p) \in P_1 \cap P_2) \leq \Pr(G(n, p) \in P_1) \cdot \Pr(G(n, p) \in P_2) \)

(c) Prove that a.s. the maximum degree in \( G \) is at most \( 10 \log n \).

(d) Prove that a.s. \( \text{dist}_G(u, v) \geq 100 \) for every two vertices \( u, v \in S \).

(e) Prove that a.s. \( |U \cup \Gamma(U)| \geq 3|U| \) for every \( U \subseteq S \).

(f) Prove that a.s. \( |U \cup \Gamma(U)| \geq 3|U| \) for every \( U \subseteq V \) of size \( n^{3/4} \leq u \leq n/4 \).