Weighted Bipartite Matching

Show that the bound for the running time of the algorithm \texttt{WeightedBipartiteMatching} presented in the lecture can be improved to $O(n^3)$.

\textit{Hint:} Prove that the running time of the inner repeat-loop can be bounded by $O(n^2)$. Observe that the sets $S$ and $T$ do not have to be calculated from scratch every time and use amortized analysis. Introduce suitable data structures to determine $\lambda$ and to update $G_\ell$ in time $O(n)$.

Hamilton Cycles in the Hypercube

Let $n \geq 2$. Show that every perfect matching in the $n$-dimensional hypercube $H_n$ can be extended to a Hamilton cycle.

\textit{Hint:} Consider the following stronger statement. Let $K(H_n)$ be the graph that is obtained from $H_n$ by adding all nonexistent edges. Show by induction on $n$ that every perfect matching in $K(H_n)$ can be extended to a Hamilton cycle by using only edges of $H_n$. 

\textbf{Discussion of the exercises on 10.04.2008.}