Graphs and Algorithms

Hamilton Paths in Tournaments

An oriented complete graph is called a tournament. Show that every tournament contains a Hamilton path, i.e., a path that visits every vertex exactly once. How many Hamilton paths are there in an acyclic tournament (a tournament without directed cycles)?

Hamilton Cycles in the Hypercube

Let $n \geq 2$. Show that every perfect matching in the $n$-dimensional hypercube $H_n$ can be extended to a Hamilton cycle.

Hint: Consider the following stronger statement. Let $K(H_n)$ be the graph that is obtained from $H_n$ by adding all nonexistent edges. Show by induction on $n$ that every perfect matching in $K(H_n)$ can be extended to a Hamilton cycle by using only edges of $H_n$. 