Exercise 1

Let $k, l \geq 2$ positive integers. Prove that

(a) $r(k, 2) = k$,
(b) $r(k, l) = r(l, k)$, and
(c) $r(k, l) \leq \binom{k+l-2}{k-1}$.

Exercise 2

(a) Prove that $r(k, l) \leq r(k - 1, l) + r(k, l - 1) - 1$ whenever $r(k - 1, l)$ and $r(k, l - 1)$ are both even.

(b) Use (a) to prove that $r(4, 3) \leq 9$.

Exercise 3

Prove Schur’s Theorem:

For every positive integer $k$ there exists a positive integer $n$ such that in any $k$-coloring of the numbers $\{1, \ldots, n\}$ there will be 3 not necessarily distinct numbers $\{x, y, z\}$ colored with the same color such that $x + y = z$.

*Hint:* Use the graph version of Ramsey’s Theorem for $r(3, 3, \ldots, 3)$.