Topics in Random Graphs

Solving this exercise sheet is not compulsory. If you would like feedback on your solution hand it in by March 25, either by email at lgugelmann@inf.ethz.ch or on paper at CAB G32.2.

Exercise 1

Prove that for all $k, \ell$ there exists a regular graph $G$ such that $\chi(G) > k$ and $\text{girth}(G) > \ell$.

Hint: Consider a random regular graph $G' \sim G_{n,r}$ for some $r = r(k)$ large enough. Bound $\alpha(G')$.

Exercise 2

Prove that for all fixed $r \geq 3$, a random regular graph $G \sim G_{n,r}$ is connected with high probability.

Remark: Using similar techniques it is possible to prove that $G$ is in fact whp $r$-connected.

Exercise 3

Let $r \geq 3$ fixed and $n$ large enough. Prove that there exists a graph $G$ on $n$ vertices with $r - 1 \leq \delta(G) \leq \Delta(G) \leq r$ such that

$$\text{girth}(G) \geq \frac{c \log n}{\log r}$$

for some constant $c > 0$.

Recall: $\delta(G)$ and $\Delta(G)$ denote respectively the minimum and maximum degree over all vertices of $G$.

Hint: Consider a random $r$-regular graph $G'$. Prove that that whp $G'$ contains no subgraph with $k$ vertices and $k + 1$ edges for all $k \leq \ldots$, i.e. that there are whp no intersecting cycles of length at most $k$. 