Graphs and Algorithms

Exercise 1

Let $G$ be a plane graph and $G^*$ its dual graph. Prove that a cycle in $G$ is a minimal (not minimum!) edge cut in $G^*$, and vice-versa.

Exercise 2

A plane graph is a triangulation if every face (including the outer face) has exactly three adjacent edges.

(a) Prove that every triangulation on at least 13 vertices has at least one vertex of degree at least 6.

Hint: Assume all vertices have degree at most 5 and bound the number of edges.

(b) Find a planar graph on six vertices, where all vertices have degree 4; or prove that no such graph can exist.

(c) Find a planar graph on six vertices and 13 edges, or prove that no such graph can exist.

(d) Let $G$ be a connected planar graph on $n \geq 1$ vertices, where all vertices have degree at least 4. Show that in any embedding of $G$ in the plane there are at least $n + 2$ faces.

Exercise 3

Show that there is a tree $T = (V, E)$ on $n$ vertices and a permutation $\pi : \{1, 2, \ldots, n\} \to V$ such that the algorithm GREEDY-COLORING($T, \pi$) needs $\Omega(\log n)$ colors.

Discussion of the solution in the exercise class on 10.5.2012.