Exercise 1 (Petersen, minors and subdivisions)
Consider the so called Petersen graph in the picture below. Prove that the Petersen graph...

a) ... contains a subdivision of $K_{3,3}$.

b) ... contains no subdivision of $K_5$.

c) ... contains both $K_{3,3}$ and $K_5$ as minors.

Exercise 2 (Outerplanar graphs)
We say that a graph $G$ is outerplanar if there exists a plane drawing of $G$ such that every vertex belongs to the outerface.

(a) Give a polynomial time algorithm which checks whether a given graph $G$ is outerplanar.

(b) Prove that every outerplanar graph is 3-colorable.

(c) Use part (b) to prove the art gallery problem: If an art gallery is laid out as a simple polygon with $n$ sides (see Figure[1] for an example), then it is possible to place $\lfloor n/3 \rfloor$ guards on the boundary or the interior of the polygon such that every point of the interior is visible to some guard.

(d) Construct an art gallery (polygon) that requires $\lfloor n/3 \rfloor$ guards.

Discussion of the solution in the exercise class on 23.05.2013.
Figure 1: Simple Polygon