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Handout 6

Randomized Algorithms and Probabilistic Methods: Advanced Topics

Exercise 1

Let $Q_1 \subset nB$ and let $Q_2 \subset n^2B \setminus nB$ with $|Q_1| + |Q_2| = poly(n)$. A convex body K is consistent (with Q_1 and Q_2) if $Q_1 \subseteq K$ and $K \cap Q_2 = \emptyset$. Let K_1 be the smallest consistent body and K_2 be the largest consistent body with $K_2 \subseteq n^2B$. Show

$$\frac{2^n}{poly(n)} \cdot vol(K_1) \leq vol(K_2).$$

Hint: Show that the convex hull CH of Q_1 is a subset of $\bigcup_{p \in CH} B_2(p/2, ||p/2||)$. In order to do that, show that if a point x is not in $B_2(p/2, ||p/2||)$ for some $p \in CH$, then p is in the open halfspace which is bounded by the hyperplane through x perpendicular to x and contains the origin.

Exercise 2

Let K be a convex body in $\mathbb{R}^n$ such that $B \subseteq K \subseteq 2^{poly(n)}B$, where B is the infinity norm ball with radius 1. Show that there exists an affine transformation f such that $B \subseteq f(K) \subseteq n^2B$ and that f can be computed in $poly(n)$ time. You may assume that you can optimize a linear function over K in $poly(n)$ time.