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Handout 7

Randomized Algorithms and Probabilistic Methods: Advanced Topics

Exercise 1

In this exercise, we prove that expander graphs are in some sense highly connected.
Prove that for every $\lambda < d$, there exists a constant C such that the following holds for every d-regular graph G on n vertices with edge expansion at most λ: if one removes $m \geq 0$ edges from G, then the resulting graph has a connected component of size at least $n - Cm$.

Exercise 2

In this exercise, we prove that expander graphs have small diameter.
Prove that for every $\lambda < d$, there exists a constant c such that for every d-regular graph G with spectral expansion λ and every vertex v of G, one has

$$|B(v,r)| \geq \min\{(1+c)^r, n\},$$

where $B(v,r)$ denotes the ball of radius r around v (i.e., the set of all vertices with distance at most r to v). *Hint:* prove first the weaker statement that $|B(v,r)| \geq \min\{(1+c)^r, n/2\}$.