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Handout 8

Randomized Algorithms and Probabilistic Methods: Advanced Topics

Exercise 1

In this exercise, you prove that the Cheeger inequality is tight up to multiplicative factors. Recall, that Cheeger's inequality states that if G is a d-regular graph on n vertices for some $n > d \geq 1$ with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$, then

$$\frac{h(G)^2}{2d} \leq d - \lambda_2 \leq 2h(G).$$

For the right side of the inequality show that there exists an infinite family of connected graphs with $d - \lambda_2 \geq 2h(G)$. For the left side show that there exists an infinite family of connected graphs with $d - \lambda_2 \in \mathcal{O}(h(G)^2/(2d))$.

Exercise 2

Show that for $d \leq 2$ there exists no family of (spectral/edge) expanders with degree d.

Exercise 3

Let $G = (V, E)$ be a d-regular graph on n vertices for some $n > d \geq 1$. Define the complement of G as $\overline{G} = (V, \binom{V}{2} \setminus E)$. Show

$$\lambda_i(\overline{G}) = -1 - \lambda_{n+2-i}(G)$$

for all $2 \leq i \leq n$, where $\lambda_i(G)$ denotes the i-th largest eigenvalue of the adjacency matrix of G.