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Handout 9

Randomized Algorithms and Probabilistic Methods: Advanced Topics

Exercise 1

The min-degree graph process is defined as follows. Start with G_0 being the empty graph on n vertices. Then, in every step, G_{m+1} is obtained from G_m by choosing uniformly at random a vertex v of minimum degree in G_m and connecting it to a randomly chosen vertex u. For simplicity, we will allow loops and multiple edges.

The goal in this exercise is to determine the time until every vertex has degree at least two. Let us denote by $X(m)$ the number of vertices in G_m with degree less than two. Determine the threshold t_0 such that

$$\begin{cases} X(tn) > 0 & \text{whp if } t < t_0, \text{ and} \\ X(tn) = 0 & \text{whp if } t > t_0. \end{cases}$$

Hint: divide the process into two phases, depending on whether there are isolated vertices or not. Use differential equations to describe the process for each phase.