

Advanced Data Structures

Spring Semester 2017

Exercise Set 6

Exercise 1:

(Dominance Query) For two points, $p = (x_p, y_p)$ and $q = (x_q, y_q)$, p is said to *dominate* q if $x_q \leq x_p$ and $y_q \leq y_p$. In Exercise Set 5, we have already considered how to process a set S of n points in the plane such that for a query point $p = (x_p, y_p)$, the points in S that are dominated by p can be answered in $\mathcal{O}(\log n + k)$ time, where k is the output size.

Now, we further consider a dynamic structure for this dominance problem. We want to add and delete a point in $\mathcal{O}(\log^2 n)$ time.

Hint: This query problem is decomposable.

Exercise 2:

In Exercise Set 5, we have already considered how to process a set S of n disjoint axis-parallel rectangles such that for a query vertical line segment l , the rectangles in S intersected by l can be answered in $\mathcal{O}(\log n + k)$ time, where k is the output size.

Now, we further consider a dynamic structure that allows the insertion and the deletion of an axis-parallel rectangle in $\mathcal{O}(\log^2 n)$ time. Of course, no two rectangles can intersect each other.

Hint: This query problem is decomposable.

Exercise 3:

(Point Location Query). Consider a planar subdivision of a rectangular area, and let the number of edges of the subdivision be n . We attempt to preprocess the planar subdivision in $\mathcal{O}(n \log n)$ time such that for a query point x inside the rectangular area, the region in the planar subdivision that contains x can be answered in $\mathcal{O}(\log n)$ time.

Moreover, we further consider a dynamic structure for this planar point location problem. We are allowed add and delete an edge, but no two edges cross each other. Each operation should take $\mathcal{O}(\log^2 n)$ time.

Hint:

- The plane-sweep paradigm.
- This query problem is decomposable.