Przemysław Uznański May 09, 2017

Advanced Data Structures

Spring Semester 2017

Exercise Set 11

Below, $\circ$ denotes concatenation.

Exercise 1:
Let our input word be $x = 0^t \circ \varepsilon_1 \circ 0^t \circ \varepsilon_2 \circ \ldots \circ 0^t \circ \varepsilon_k$, where $\varepsilon_i \in \{0, 1\}$ (assume $t \gg \log w$). Show that there is constant m, such that we can extract $\texttt{bitcount}(x)$ from $(x \cdot m)$ using $\mathcal{O}(1)$ bit operations.

Exercise 2:
Parallel search in $\mathcal{O}(1)$ bit operations. That is, show that given $s_1 < s_2 < \ldots < s_k$ of *the same length*, we can pack them into single word (assume you have enough space), that given q of the same length as s_i, we can find using constant number of operations, i such that $s_i \leq q < s_{i+1}$.

Hint: Assume you have some extra space (this is not a problem) and pack into single word $a_1 = 1 \circ s_1 \circ 1 \circ s_2 \circ \ldots \circ 1 \circ s_k$ and consider $a_2 = 0 \circ q \circ 0 \circ q \circ \ldots \circ 0 \circ q$. Use Exercise 1 on $a_1 - a_2$.

Exercise 3:
Most significant '1' on short inputs: given x such that $|x| = \sqrt{w}$, show that we can find position of the most significant '1' in x.

Hint: Use parallel search of x among $\sqrt{w}$ words of length $\sqrt{w}$.

Exercise 4:
Most significant '1' on inputs as in Exercise 1: $x = 0^k \circ \varepsilon_1 \circ 0^k \circ \varepsilon_2 \circ \ldots \circ 0^k \circ \varepsilon_k$ where $k = \sqrt{w}$.

Hint: There is explicitly given constant m, such that $(x \cdot m)$ contains a single block of bits: $\varepsilon_1 \varepsilon_2 \ldots \varepsilon_k$.

Exercise 5:
Most significant '1' on any input.

Hint: Cut your input into $\sqrt{w}$ blocks of length $\sqrt{w}$ (mentally, we cannot aford it). With little work, we can find first non-empty block (and reduce that part to Exercise 4). Inside first non-empty block, use Exercise 3.