

Exercise Set 11 – FS18

(Linear Algebra Methods in Combinatorics)

You can submit solutions **also by email** by the next lecture – **24.5.2018**. These exercises are **non-graded** but you get feedback on your submitted solutions.

Two exercises on disproving Borsuk conjecture for a small dimension.

Exercise 1. A **strongly regular** graph with parameters $(n, \delta, \lambda, \mu)$ is an undirected graph with n nodes, each node has degree δ and such that

1. Any two neighbors have λ common neighbors;
2. Any two non-neighbors have μ common neighbors.

Show that, for any strongly regular graph with parameters as above, you can construct a **spherical two-distance** set in \mathbb{R}^n , that is, a subset of points in the sphere $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : \|x\| = 1\}$ that for a two-distance set.

Exercise 2. There exists a graph with parameters $(416, 100, 36, 20)$ which does not contain any clique of size 6. Consider the diameter graph G_B derived from the two-distance set in the previous exercise. Prove the following:

- There exists a spherical two-distance set in \mathbb{R}^{400} which requires at least $n/5 = 416/5$ pieces to reduce its diameter.