Exercise Set 4 (Linear Algebra Methods in Combinatorics – HS11)

These exercises will be graded. Please return the solutions at the beginning of next lecture – 26.10.2011. You can send solutions also by email (same deadline).

Exercise 1 (2 points). Show that the following construction of $t$-Ramsey graphs based on two-distance sets is wrong:

I construct a two-distance set $s_1, \ldots, s_m \in \mathbb{R}^t$ and from this I consider the graph with vertices corresponding to these points. I color the edges according to the Euclidean distance between the points:

$$\text{color}(s_i, s_j) = d(s_i, s_j) \in \{\delta_1, \delta_2\}$$

So my two colors are “red $= \delta_1$” and “blue $= \delta_2$”.

Explain why I cannot guarantee that this is a $t$-Ramsey graph.

Exercise 2 (3 points). Extend the proof of the Ramsey Theorem for graphs given in the lecture to the case of any number $c$ of colors:

**Theorem.** For every natural numbers $c$ and $t$, there exists a natural number $n = R_c(t)$ such that, if we color the complete graph with $n$ or more nodes using $c$ colors, then there must be monochromatic complete subgraph of size $t$.

If you cannot prove the theorem above, then try to prove the case $c = 3$ (2 points in this case).

Note: a graph with just one node is by definition a monochromatic component; this may help for the base case of the induction (look back at the proof for two colors).

Exercise 3 (5 points). Give a construction of a 3-coloring of the complete graph with $n = \binom{t}{3}$ nodes so that there is no monochromatic complete subgraph of size $t + 1$ (try to adapt the idea of the cubic construction).

Note: by “3-coloring” we mean that we color the edges using colors red, blue, and green.