Exercise 4.1  Advanced Search Trees.

In this exercise, we wish to extend the functionality of a search tree. We consider a binary search tree over integers. In addition to finding a number, we want to be able to answer the following questions:

a) How many elements in the tree are even and smaller than a given number $k$?

b) How many even elements in the tree are (strictly) between two given numbers $k_1$ and $k_2$, with $k_1 < k_2$?

Discuss how you can modify the tree so that you can answer these questions efficiently. Describe how the insertion and the removal operation must be changed accordingly. Include a discussion of the running times of the modified algorithms.

Exercise 4.2  Traversal Rules for Trees.

a) Give the sequence generated by a pre-order traversal of the following binary search tree:

```
  9
 / \    
 5   20
 /    /
4  7  13
 //  //
2  6  8
 |   |
1   3
```

b) Draw the binary search tree that generates the following post-order traversal: 2, 4, 3, 7, 6, 10, 9, 8, 5.
Exercise 4.3  *AVL-Trees (Part of an Exam in August 2010).*

Insert in the following AVL-tree the key 6, and delete from the resulting AVL-tree the key 23.

![AVL Tree Diagram]

<table>
<thead>
<tr>
<th>After insertion of 6:</th>
<th>After deletion of 23:</th>
</tr>
</thead>
</table>

Exercise 4.4  *Blum’s Median-of-Median Strategy.*

We consider finding the median of a sequence using the median-of-median strategy from the lecture (see Chapter 3.1 in the book). We will consider only the highest level of recursion, so only the very first invocation of the procedure “Auswahl” that determines the $i$-th smallest element with $i = \lceil \frac{N}{2} \rceil$.

a) Given the following sequence

$$7, 12, 17, 3, 10, 1, 6, 2, 4, 8, 11, 9, 9, 6, 5, 14, 20, 13, 1, 7, 19, 8,$$

provide the two sequences on which Auswahl invokes itself recursively.

b) In general, how long at least and at most are each of the two sequences used in the two recursive calls of the procedure Auswahl for $i = \lceil \frac{N}{2} \rceil$?

**Hand-in:** until Wednesday 21st March 2012.