Exercise 6.1  Median-of-three Quicksort.

We use quicksort with the median-of-three strategy (see the book, section 2.2.2 for a detailed description) to sort $A[l..r]$: First, we calculate the median of the elements $a_l$, $a_m$ and $a_r$ where $m = \lfloor (l+r)/2 \rfloor$, then we swap $a_r$ with the median and use $a_r$ as the pivot. Given $n_0$, provide a sequence of length $n \geq n_0$ for which the median-of-three quicksort compares $\Omega(n^2)$ keys. Prove that this sequence really leads to a quadratic number of key comparisons.

Exercise 6.2  Number of different Search Trees.

Let $K_n = \{1, 2, ..., n\}$ be a set of keys. Derive a recursive formula for the number of different binary search trees that contain exactly the keys in $K_n$. You do not need to eliminate the recursion.

Exercise 6.3  Traversal of Trees.

a) Give the sequence generated by a preorder and a postorder traversal of the following tree.

![Tree Diagram]

b) Draw the binary search tree that generates the preorder traversal 11, 5, 9, 21, 14, 12, 19, 17, 22, 30.

Exercise 6.4  Hash functions.

We consider open hashing with a hash table of size $p$ for a prime $p$.

a) Decide which of the following functions are useful as a hash functions (and which are not), and justify your answer.

- $h(k) = \text{Digit sum of } k$
- $h(k) = k(1 + p + p^2) \mod p$
- $h(k) = \lfloor p(rk - \lfloor rk \rfloor) \rfloor$, $r \in \mathbb{R}^+ \setminus \mathbb{Q}$

b) In this task we use the hash function $h(k) = k \mod p$ and resolve collisions using double hashing. Let $q$ be the largest prime smaller than $p$, $h'(k)$ the second hash function and $s(j, k)$ the probing function. The complete hash function in the $j$-th step is $h(k) - jh'(k) \mod p$ if $h'(k)$ is given, and $h(k) - s(j, k) \mod p$ if $s(j, k)$ is given. Decide which of the following choices of $h'(k)$ and $s(j, k)$ are reasonable (and which are not), and justify your answer.

- $h'(k) = \lceil \ln(k + 1) \rceil \mod q$
- $s(j, k) = k^j \mod p$
- $s(j, k) = ((k \cdot j) \mod q) + 1$

**Hand-in:** until Wednesday, 10th April 2013.