Exercises.

1) Insert the key 15 in the AVL tree below. After that, remove the key 3 from the resulting tree.

![AVL Tree Diagram]

After insertion of 15:

After deletion of 3:

2) The following array contains the elements of a max-heap stored in the usual fashion. Specify the array after the maximum has been removed and the heap condition has been reestablished.

```
27 17 20 15 7 9 13 8 2 5 3 1 6
1 2 3 4 5 6 7 8 9 10 11 12 13
```
3) Use open hashing to insert the keys 11, 15, 23, 21, 8, 16 in this order into the following hash table. Use Double Hashing with the hash function \( h(k) = k \mod 7 \) and resolve collisions using the function \( h'(k) = 1 + (k \mod 5) \).

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<table>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

4) Specify (as concisely as possible) the asymptotic running time of the following code fragment in \( \Theta \) notation depending on \( n \in \mathbb{N} \). You do not need to justify your answer.

```java
for(int i = n; i > 1; i = i/2) {
    for(int j = 1; j <= n; j++)
    ;
}
```

5) Consider the following recursive formula:

\[
T(n) := \begin{cases} 
6 + 7T(n/3) & n > 1 \\
6 & n = 1 
\end{cases}
\]

Specify a closed form (i.e., non-recursive) for \( T(n) \) that is as simple as possible, and prove its correctness using mathematical induction.

**Hints:**

(1) You may assume that \( n \) is a power of 3.

(2) For \( q \neq 1 \), we have \( \sum_{i=0}^{k} q^i = \frac{q^{k+1}-1}{q-1} \).