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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 1

Exercise 1: (3 + 1 Points)
Consider the following normal form, minimization game between two players, "row" and "column." Both players have four strategies $S_{row} = S_{col} = \{A, B, C, D\}$. For each strategy profile $(s_{row}, s_{col}) \in S \times S$ the entry in the lower left corner of the corresponding cell in the table below denotes the cost to the row player and the entry in the upper right corner the cost to the column player.

	A	B	C	D
A	6 4	5 6	4 5	5 6
B	6 5	5 1	1 2	5 5
C	6 6	3 5	3 3	2 1
D	6 7	3 7	5 6	7 7

a) Find all pure Nash equilibria of this game. Explain why exactly those states are pure Nash equilibria and why no other state is one.

b) Find all mixed Nash equilibria of this game. Explain why these probability distributions are the only mixed Nash equilibria.

Exercise 2: (5 Points)
Consider the following normal form, maximization game. Each of $n \geq 2$ players announces a number in $\{1, \ldots, k\}$. A prize of 1'000 CHF is split equally among the players whose chosen number is closest to 2/3 times the average number. Show that this game has a unique Nash equilibrium and that in this equilibrium each player picks a pure strategy.

Hint: Think about strategies that you can exclude.

Exercise 3: (2 + 4 + 4 + 6 + 1 + 4 Points)
Consider the following normal form, maximization games, Game 1 and Game 2, between two players, "row" and "column." In both cases, the strategy spaces of the two players are $S_{row} \subseteq \{T, M, B\}$ and $S_{col} = \{L, R\}$. For each strategy profile $(s_{row}, s_{col}) \in S_{row} \times S_{col}$ the entry in the lower left corner of the corresponding cell in the two tables below denotes the payoff to the row player and the entry in the upper right corner the payoff to the column player.

	L		R	
T		6		5
	2		6	
B		4		6
	4		2	

Game 1

	L		R	
T		6		6
	3		3	
M		2		2
	4		4	
B		1		6
	3		5	

Game 2

For each of the two games:

(a) Find all pure Nash equilibria of this game. Explain why exactly those states are pure Nash equilibria and why no other state is one.

(b) Find all mixed Nash equilibria of this game. Explain why there can be no other mixed Nash equilibria.

(c) True or false? From any start state (s_{row}, s_{col}) any sequence of (best response) improvement steps converges. Either prove that this is the case or give a counter example.