Paul Dütting, ETH Zürich October 26, 2015
Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 6

Exercise 1: (6 Points)
Let $G = (V, E)$ be a 2-edge-connected graph, i.e., between any two nodes of the graph there exist at least two edge-disjoint paths.
Suppose that G represents a communication network and that every edge e represents a link which is owned and operated by an agent A_e. A company is interested in buying a subset T of the links at the lowest possible cost, such that every pair of nodes in the graph G can communicate along edges of T. That is, the company wants to buy a set of edges T that induces a *spanning tree* of G.
Each agent A_e has a cost t_e for operating the link, which is not known to the company. The company therefore asks each agent for his cost, to which the agents reply with reported costs c_e. It then selects a spanning tree and pays each selected agent a possibly different amount p_e. Agents want to maximize their utility, which is payment minus cost, i.e., $p_e - t_e$, if they are selected and 0 otherwise.
Design a polynomial-time mechanism that induces the agents to truthfully report their costs. The utility of each agent should be non-negative. Specify how the mechanism selects T and what it pays to each agent $e \in T$. Prove that it is truthful.

Exercise 2: (4 Points)
Consider the mechanism design problem for the algorithmic problem of scheduling jobs on related parallel machines as presented in the lecture.
Recall that there are n machines (the players), each having a speed s_i with which it can process jobs. So $t_i = \frac{1}{s_i}$ is the time that machine i needs to process a job of length one. Here, t_i is the private information of the player. There are m jobs with load $l_1, l_2, \ldots, l_m$, which need to be assigned to the machines.
A mechanism asks the machines to report the time r_i that they need to process a job of length one. It then computes an assignment of jobs to machines and a payment p_i to every machine i. We use $W_i(r)$ (or W_i only, when the reports r are clear from the context) for the total load of jobs assigned to machine i under reports r. We call W_i the work of machine i. The total cost of machine i is $W_i \cdot t_i$.
Show that the following algorithm can be implemented by a (non poly-time) truthful mechanism. Fix an arbitrary order of the n machines. Select the job allocation that minimizes the makespan $\max_i W_i \cdot t_i$ whose workload on machines, when seen as a vector $(W_1, W_2, \ldots, W_n)$, is lexicographically smallest.

Exercise 3: (6 Points)
In this exercise you are asked to prove an alternative characterization of truthful mechanisms, which is stated in the following theorem. It concerns mechanisms consisting of an outcome rule f and a payment rule p.

Theorem. A mechanism is truthful if and only if it satisfies both the following conditions:

1. For any two v_i, v_i' that result in the same chosen outcome $a = f(v_i, v_{-i}) = f(v_i', v_{-i})$, the payment does only depend on the outcome. Formally, for every v_{-i}, for every $a \in A$, there exist prices $p_a \in \mathbb{R}$ such that for every v_i with $f(v_i, v_{-i}) = a$ the payment is $p_i(v_i, v_{-i}) = p_a$.

2. The mechanism optimizes for each player. Formally, for every v_i and every v_{-i} we have that
$$f(v_i, v_{-i}) = \arg\max_a (v_i(a) - p_a),$$
where the outcomes a are from the range of $f(\cdot, v_{-i})$.