Paul Dütting, ETH Zürich November 2, 2015
Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 7

Exercise 1: (6 Points)
Consider a single-parameter mechanism design problem, in which each player's type corresponds to a value. In other words, we are given a set $\mathcal{N}$ of n players, the set of feasible outcomes is X, where x_i corresponds to player i's allocation, and player i's value for outcome x is $v_i \cdot x_i$. Suppose that allocation rule f maximizes social welfare $\sum_i v_i \cdot x_i$ over feasible allocations $x \in X$. Let x^* denote the welfare-maximizing allocation for b. Consider payments

$$p_i(b_i, b_{-i}) = \max_{x \in X} \sum_{j \neq i} b_j \cdot x_j - \sum_{j \neq i} b_j \cdot x_j^*.$$

Argue that the resulting mechanism $M = (f, p)$, called the VCG mechanism, is dominant strategy incentive compatible.

Hint: The lecture notes for Week 6 & 7 prove this claim using Myerson's lemma. This exercise asks you to prove this claim directly by showing that truthful bidding is a weakly dominant strategy for each player.

Exercise 2: (6 Points)
Consider the following generalization of the sponsored search problem. In addition to a private value v_i, each bidder $i \in [n]$ now has a publicly known quality γ_i. As usual each position j has a click-through rate α_j, and $\alpha_1 > \alpha_2 \geq \cdots \geq \alpha_k > 0$. We assume that if bidder i is placed in position j, its probability of a click is $\gamma_i \cdot \alpha_j$. In other words, bidder i's value for this outcome is $v_i \cdot \gamma_i \cdot \alpha_j$.

(a) Describe the welfare-maximizing allocation rule in this generalized sponsored search setting. Argue that this rule is monotone.

(b) Give an explicit formula for the per-click payment of each bidder that extends this allocation rule to a DSIC/truthful mechanism.

(c) Draw a graph similar to the ones you saw in the lecture which visualizes this payment.

Exercise 3: (4 Points)
In a knapsack auction, each bidder i has a publicly known size w_i (e.g., the duration of a TV ad) and a private valuation (e.g., a company's willingness-to-pay for its ad being shown during a break of a Super League match). The seller has a capacity W (e.g., the length of a commercial break). We assume, without loss of generality, that $w_i \leq W$ for every i. The feasible set X is defined as the $0-1$ n-vectors $(x_1, ..., x_n)$ such that $\sum_{i=1}^n w_i x_i \leq W$. As usual, we use $x_i = 1$ to indicate that i is a winning bidder.

Consider the following algorithm for this problem:

Greedy Algorithm

1. Sort and re-index the bidders so that $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \frac{b_3}{w_3} \geq \cdots \geq \frac{b_n}{w_n}$.
2. Pick winners in this order until one doesn't fit, and then halt.
3. Return either the Step 2. solution, or the highest bidder, whichever creates the higher social welfare.

Answer the following questions:

(a) Either look up or recall from your undergraduate algorithms class that the greedy algorithm yields a 2-approximation to the optimal social welfare.

(b) Prove that the greedy algorithm can be implemented by a truthful mechanism, i.e., show that it is monotone.

Hint: You do not need to hand in an answer for (a).