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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 9

Exercise 1: (4 Points)

Another useful technique for designing truthful multi-parameter mechanisms are so-called *maximal-in-range* (or MIR) mechanisms. These mechanisms consist of a an allocation rule that computes the welfare-maximizing outcome over the restricted set $X' \subseteq X$ (its range) with respect to the reported valuations. It then charges each player his VCG payment with respect to this allocation rule.

(a) Show that MIR mechanisms are dominant strategy incentive compatible or truthful.

(b) Argue that the greedy mechanism that we developed in Exercise 3 on Exercise Sheet 7 is not an MIR mechanism.

Exercise 2: (8 Points)

Consider the following variation of a combinatorial auction. There is a set M of m items and a set $\mathcal{N}$ of n bidders. Each bidder i has a valuation function $v_i : 2^M \to \mathbb{R}_{\geq 0}$. The valuations are subadditive. That is, for all bidders i and all bundles $S, T \subseteq M$ we have $v_i(S) + v_i(S) \geq v_i(S \cup T)$. The goal is to approximate the maximum social welfare $\max_{\text{feasible } S} \sum_{i \in \mathcal{N}} v_i(S_i)$.

The problem of maximizing welfare with subadditive bidders is NP-hard, so we consider the following approximation mechanism.

The Mechanism

1. Query each bidder i for $v_i(M)$, and for $v_i(j)$, for each item $j \in M$.
2. Construct a bipartite graph by defining a vertex a_j for each item j, and a vertex b_i for each bidder i. Let the set of edges be $E = \cup_{i \in \mathcal{N}, j \in M}(a_j, b_i)$. Define the weight of each edge (a_j, b_i) to be $v_i(j)$. Compute the maximum weight matching P in this graph.
3. If the valuation of the bidder i that maximizes $v_i(M)$ is higher than the value of P, allocate all items to i. Otherwise, for each edge $(a_j, b_i) \in P$ allocate the j-th item to the i-th bidder.
4. Let each bidder pay his VCG price.

Your task is to establish that this mechanism is truthful and yields a near-optimal outcome. More specifically:

(a) Argue that this mechanism is dominant strategy incentive compatible or truthful.

(b) Show that this mechanism achieves a $O(\sqrt{m})$-approximation to the optimal social welfare.

Hint: For (a) the first exercise on this sheet may be helpful. For (b) it may be useful to consider the optimal allocation and distinguish between small bundles with size $< \sqrt{m}$ and large bundles with size $\geq \sqrt{m}$.

Exercise 3: (4 Points)
In the lecture we have developed a truthful mechanism for the combinatorial auction problem with k-minded bidders. This mechanism was based on the greedy by square root of bundle size algorithm. I claimed that no payment rule can turn this allocation rule into a truthful mechanism. This exercise asks you to prove this claim.

Specifically, we will consider the following simpler version which ranks by value divided by size. All arguments can be extended to also apply to the case where we rank by value divided by square root of the bundle size.

The Mechanism

1. Rank all bidder bundle pairs (i, S) by non-increasing score $v_i(S)/|S|$.
2. Go through the sorted list of bundles and assign the next bundle to he respective bidder unless
   (a) this bidder has already received a different bundle, or
   (b) some item in the desired bundle has already been assigned to a different bidder.

Use the following example to show that no payment scheme can turn this allocation rule into a truthful mechanism:

- There are two goods a and b and two bidders Green and Red.
- Red is single-minded and he has a value of 10 for all allocations in which he gets a. Red declares his type truthfully.
- Green is interested in both b and the set $\{a, b\}$. His valuation is 0 for any allocation that does not contain b. It is v_b for any allocation that contains b but not a and it is v_{ab} for any allocation in which he receives both items. Green's declaration is 0 for all allocations that do give him b, it is $0 \leq g_b < 10$ for all allocations in which he receives b but not a, and it is g_{ab} for the allocation in which he gets both items.

Hint: Reason by contradiction and assume a truthful payment scheme exists. Then make a case distinction whether $g_{ab} > 20$ or $g_{ab} < 20$. What is the payment allowed to depend upon? What does this imply? Then assume Green is bidding truthfully and distinguish between the case $v_{ab} < 20$ and $v_{ab} > 20$. What are the requirements for truthfulness? Are they satisfied for all possible parameters?