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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 10

Exercise 1: (4 Points)
Re-consider the single-item first-price auction discussed in the lecture. Show that this mechanism is actually $(1 - 1/e, 1)$-smooth by considering the highest value bidder and the randomized deviation in which this bidder draws a bid from $[0, (1 - 1/e) \cdot v_h]$, where v_h is this bidders value, with density $f(x) = 1/(v_h - x)$.

Exercise 2: (4 Points)
Now consider a variant of the single-item auction in which all players have to pay their bid, even if they loose. Show that this all-pay auction is $(1/2, 1)$-smooth by considering the highest value bidder and the randomized deviation in which this bidder draws a bid uniformly at random from $[0, v_h]$, where v_h is this bidders value.

Exercise 3: (6 Points)
Consider the Generalized First-Price Mechanism (GFP) for sponsored search. This mechanisms solicits a bid b_i from every bidder i, allocates bidders so that the bidder with the highest value is allocated the first position, the bidder with the second highest value the second highest, and so on. A bidder i that is assigned position j has to pay $\alpha_j \cdot b_i$, while bidders that are not allocated any position pay nothing. Show that the GFP mechanism is $(1 - 1/e, 1)$-smooth.

Hint: The argument with which you can show smoothness in this case is very similar to the one in Exercise 1.

Exercise 4: (6 Points)
In the lecture, when showing that combinatorial auctions with item bidding are $(1/2, 1)$-smooth for submodular valuations, we used that every submodular function can be approximated with an additive bid. That is, for every submodular function v and bundle T there exists an additive function a such that

$$\sum_{j \in S} a(j) \begin{cases} = v(T) & \text{for } S = T \text{, and} \\ \leq v(S) & \text{otherwise.} \end{cases}$$

Prove this lemma.