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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2015

Exercise Set 11

Exercise 1: (3 Points)
Consider a sponsored search auction with three slots with click-through rates $\alpha_1 = 1.0$, $\alpha_2 = 0.6$, and $\alpha_3 = 0.2$ and four bidders with values $v_1 = 24$, $v_2 = 14$, $v_3 = 6$, and $v_4 = 2$.

1. What is the truthful VCG outcome?
2. What is the highest revenue symmetric pure Nash equilibrium?
3. What is the lowest revenue symmetric pure Nash equilibrium?

For each of the three cases specify the allocation and payments as well as the overall social welfare and revenue.

Exercise 2: (5 Points)
A desirable property in sponsored search practice is that prices are decreasing with slots. That is, higher slots have higher prices per click and in total. Show that symmetric pure Nash equilibria have this property. That is, show that in every symmetric pure Nash equilibrium it holds that

$$\alpha_{s-1} p_{s-1} \geq \alpha_s p_s \qquad \text{and} \qquad p_{s-1} \geq p_s \qquad \text{for all } s.$$

Exercise 3: (4 Points)
Search engines can observe the equilibrium prices, but typically not the values that the bidders have. Show that both pure Nash equilibria and symmetric pure Nash equilibria allow to recover partial information about the unknown values.

(a) Derive lower and upper bounds on the value v_s of the bidder that assigned to slot s in a pure Nash equilibrium.

(b) Do the same for symmetric pure Nash equilibrium.

Exercise 4: (* Points)
(Bonus) Edelman et al. define a pure Nash equilibrium of the GSP mechanism as "locally envy free" if no bidder wants to exchange bids with the player ranked one slot above him. How does this competing definition relate to the definition of a symmetric pure Nash equilibrium. Prove your claim.

Hint: For the formal definition and additional background look up the paper by Edelman et al. cited in the lecture notes.