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Deadline: Beginning of next lecture

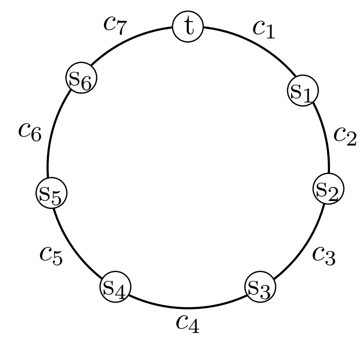
Algorithmic Game Theory

Fall 2016

Exercise Set 4

Exercise 1: (3 Points)

Consider the following fair cost-sharing game:

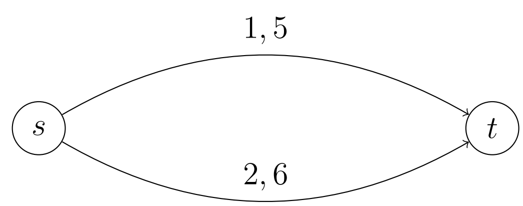


Each player i wants to be connected from s_i to t, and the edges are **undirected**. That is, two or more players can use the same edge e in different directions, and they all share equally the cost c_e of this edge.

Prove that the price of stability for pure Nash equilibria is at most 2 in this game. (In fact, you can consider any ring with n players.)

Exercise 2: (1+2+1 Points)

Consider this symmetric network congestion game with two players:



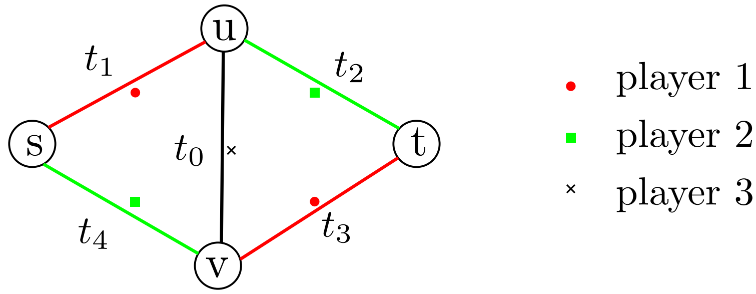
(a) What are the price of anarchy and the price of stability for pure Nash equilibria?

(b) What are the price of anarchy and the price of stability for mixed Nash equilibria? **Hint:** Start by listing all mixed Nash equilibria. To obtain these start with a sentence like, "*Let σ be a mixed Nash equilibrium with* $\sigma_1 = (\lambda_1, 1 - \lambda_1)$, $\sigma_2 = (\lambda_2, 1 - \lambda_2)$," and continue by deriving properties of λ_1 and λ_2.

(c) What is the best price-of-anarchy bound that can be shown via smoothness?

Exercise 3: (2+2+1 Points)

Consider the following variant of the shortest path mechanism design problem. Each player may own more than one edge:

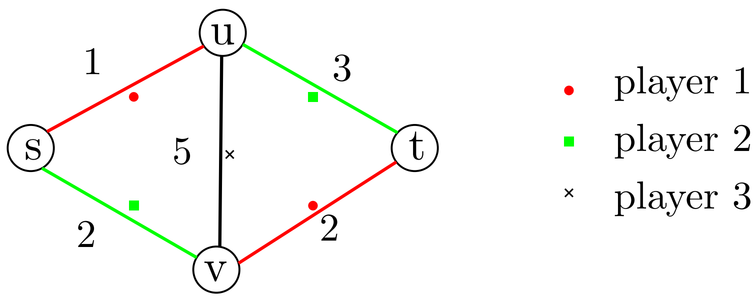


The feasible solutions consist of the paths connecting s to t, and the cost for a player is given by the sum of the costs of his/her edges in the chosen path. For example, the path s → u → v → t has costs to player 1 equal to $t_1 + t_3$, while s → u → t costs him/her t_1.

1. Describe a truthful mechanism computing the shortest path in this graph. Note that the mechanism knows which edges belong to which player. A player can misreport the cost of his/her edges.

2. Make sure that your mechanism on this particular graph satisfies **voluntary participation**: if a player is truth-telling, then his/her utility is non-negative (prove it).

3. Write the payments of your mechanism when the reported/true costs are as follows:



Exercise 4: (3 Points)

Consider the following mechanism design problem. We have three jobs of size J_1, J_2, J_3, and two machines of cost t_1 and t_2, respectively. The players are the machines: player i has cost $load_i \cdot t_i$, where $load_i$ is the sum of the jobs size allocated to this machine, and players can report a cost $c_i \neq t_i$.

Suppose we allocate jobs using a **greedy algorithm** $A_{greedy}(c_1, c_2)$:

1. The current job J_k is allocated to machine 1 if

$$(load_1 + J_k) \cdot c_1 \leq (load_2 + J_k) \cdot c_2$$

where $load_1$ and $load_2$ are the current loads of the two machines (the loads given by the jobs allocated so far).

2. Else J_k is allocated to machine 2.

Prove that there is no truthful mechanism (A_{greedy}, P).
Hint: Consider jobs of size $J_1 = 3$, $J_2 = 2$, $J_3 = 2$, and two input costs, $(c_1, c_2) = (1, 1 - \epsilon)$ and $(c'_1, c'_2) = ((1 - \epsilon)^2, 1 - \epsilon)$.