A. Mamageishvilli, P. Penna, P. Uznanski

October 24, 2016
Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2016

Exercise Set 5

Exercise 1: (1+1+1 Points)
Consider an auction with n bidders (players) and k **identical items**. If bidder i gets g_i copies of the item, then his/her valuation is $g_i \cdot v_i$, where v_i is a **private valuation** for a single copy. Each bidder reports some b_i and, based on all bids, the items are allocated to the bidders and each player is charged some amount of money.

Adapt the monotonicity condition for truthfulness described in the lecture to this setting, and show the following:

1. A mechanism (A, P) is truthful only if A is monotone (explain what monotone means for this problem);
2. Given any monotone algorithm A as above, describe the payment function P such that (A, P) is truthful.
3. Show that your mechanism satisfies voluntary participation, and bidders that do not get any item do not pay anything.

Exercise 2: (4 Points)
In a knapsack auction, each bidder i has a publicly known size w_i (e.g., the duration of a TV ad) and a private valuation v_i (e.g., a company's willingness-to-pay for its ad being shown during a break of a Super League match). The seller has a capacity W (e.g., the length of a commercial break). We assume, without loss of generality, that $w_i \leq W$ for every i. The feasible set X is defined as the $0-1$ n-vectors $(x_1, \ldots, x_n)$ such that $\sum_{i=1}^{n} w_i x_i \leq W$. As usual, we use $x_i = 1$ to indicate that i is a winning bidder.

Consider the following algorithm for this problem:

Greedy Algorithm

1. Sort and re-index the bidders so that $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \frac{b_3}{w_3} \geq \cdots \geq \frac{b_n}{w_n}$.
2. Pick winners in this order until one doesn't fit, and then halt.
3. Return either the Step 2. solution, or the highest bidder, whichever creates the higher social welfare.

Answer the following questions:

(a) Either look up or recall from your undergraduate algorithms class that the greedy algorithm yields a 2-approximation to the optimal social welfare.

(b) Prove that the greedy algorithm can be implemented by a truthful mechanism, i.e., show that it is monotone.

Hint: You do not need to hand in an answer for (a).

Exercise 3: (2 Points)
Prove that the problem of scheduling selfish related machines admits a truthful mechanism (prove Theorem 5 in the lecture notes). That is, consider the following exact algorithm. Given the set of all allocations, sorted in some order

$$\mathcal{A} = \{a^1, a^2, \ldots, a^N\}$$

the algorithm returns the first (lexicographically minimal) allocation among those minimizing the makespan with respect to the input costs $c_1, \ldots, c_n$. That is, it picks the minimum s such that

$$makespan(a^s, c) \leq makespan(a^h, c)$$

for all h and return solution a^s. Show that this algorithm A is monotone.

Exercise 4: (2 Points)
Show that VCG mechanisms yield an n-approximate truthful mechanism for scheduling n unrelated machines (makespan minimization).

Exercise 5: (2 Points)
Prove that for two unrelated machines there is no truthful mechanism wich is α-approximate for some $\alpha < 2$ (prove Corollary 7 in the lecture notes).