A. Mamageishvilli, P. Penna, P. Uznanski October 31, 2016
Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2016

Exercise Set 6

Exercise 1: (2 Points)
Consider the following mechanism (A, P) which uses a **generic algorithm** A together with payments of the VCG mechanism. On input bids $b = (b_1, \ldots, b_i, \ldots, b_n)$ do the following:

(i) Compute allocation $A(b)$;

(ii) Charge each bidder i an amount

$$P_i(b) := Q_i(b_{-i}) - \left(\sum_{j \neq i} b_j(A(b))\right) \tag{1}$$

where $Q_i()$ is an arbitrary function independent of b_i.

Prove that (A, P) above is truthful if and only if algorithm A satisfies the following condition: for every b, for every i, and every $b'_i \neq b_i$

$$SW(A(b), b) \geq SW(A(b'_i, b_{-i}), b) \ , \tag{2}$$

where $SW(a, b) := \sum_i b_i(a)$ is the social welfare of allocation a with respect to bids b.

Note: Recall that we are in the setting where each bidder i has a private valuation function $v_i()$ and therefore his/her utility in this mechanism is $v_i(A(b)) - P_i(b)$.

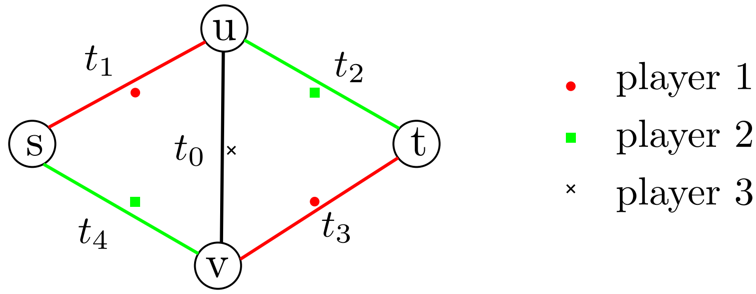
Exercise 2: (2 Points)
Consider the single minded combinatorial auction discussed in the lecture: each bidder i has a public subset S_i^* of items that he/she is willing to buy, and a **private** valuation v_i^* for this subset.
Suppose we run the mechanism (A, P) described in the previous exercise, with A being the **Greedy-by-value** algorithm. Prove that this mechanism is **not** truthful.

Exercise 3: (1+1 Points)
Suppose (A, P) is truthful. Show that A must satisfy the following monotonicity condition: for all v, for all i and for all v'_i,

$$v_i(a) + v'_i(a') \geq v_i(a') + v'_i(a)$$

where $a = A(v)$ and $a' = A(v'_i, v_{-i})$.
Show that this is a generalization of the monotonicity condition for the one-parameter setting (k identical items in previous exercise sheet, and single minded bidders).

Exercise 4: (2 Points)
Reconsider the following variant of the shortest path mechanism design problem in a previous exercise sheet. Each player may own more than one edge:



The feasible solutions consist of the paths connecting `s` to `t`, and the cost for a player is given by the sum of the costs of his/her edges in the chosen path. For example, the path $\mathsf{s} \to \mathsf{u} \to \mathsf{v} \to \mathsf{t}$ has costs to player 1 equal to $t_1 + t_3$, while $\mathsf{s} \to \mathsf{u} \to \mathsf{t}$ costs him/her t_1.

Consider the mechanism which computes the shortest path as usual, and pays each player i the cost of the best alternative path, that is, the length of the shortest path in the graph in which all edges of player i are removed. (For example, for $i = 3$, this is the shortest path in the graph where the black edge is removed.)

Prove that this mechanism is **not** truthful (already in the graph in the figure above).