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# Algorithmic Game Theory

Fall 2016

Exercise Set 12

**Exercise 1:** (4 Points)
Consider the Interns-Hospitals (Stable Matching) problem with **two interns** (1 and 2) and **two hospitals** ($A$ and $B$) where now hospitals can have **different preferences** over the interns ($\prec_A$ and $\prec_B$).

Prove that for every hospital preferences ($\prec_A$ and $\prec_B$) there is a mechanism which is obviously strategyproof for the interns.
**Note:** When the hospital preferences coincide ($\prec_A = \prec_B$) we have already a mechanism – the one in the lecture notes. Here you have to design a mechanism for every possible preferences of the hospitals (which are assumed to be public).

**Exercise 2:** (2+2 Points)
Consider the cost-sharing mechanism described in the lecture notes:

**Cost-sharing Mechanism $M_\sigma$**

1. Start with the set $S$ of all players;
2. If there is a player $i$ in $S$ with $b_i < \sigma_i(S)$ then drop $i$ from $S$; Repeat this step, in any order, until all $i$ in $S$ satisfy $b_i \geq \sigma_i(S)$.
3. Service all players in $S$ and charge each $i \in S$ an amount $\sigma_i(S)$.

Consider cross monotonic cost-sharing methods $\sigma$: for any $S, S'$ with $S \subset S'$ it holds $\sigma_i(S) \geq \sigma_i(S')$ for all $i \in S$.

Your task:

1. Show that the order in which we drop a user in Step 2 is not relevant: the output of the mechanism will be always the same.
2. Give a direct proof that $M_\sigma$ is truthful.

**Exercise 3:** (2 Points)
In the lecture notes we used the mechanism $M_\sigma$ for cost-sharing on **fixed trees**. Here we consider the generalization to **arbitrary networks**: we are given a weighted graph where every edge $e$ has some cost $C_e$ and a node $s$ representing the server. The cost $C(S)$ for servicing a subset $S$ of the nodes is the cost of the cheapest tree containing all nodes in $S$ and the server $s$.

Show that the mechanism $M_\sigma$ where $\sigma$ is the fair cost-sharing method is **not truthful** for this version of the problem.

**Note:** In this mechanism the cost-sharing method $\sigma$ is computed as follows. For each subset $S$ of players, a corresponding optimal tree $T(S)$ of minimal cost is computed. Then the cost of each edge $e \in T(S)$ is shared equally among all $i \in S$ whose path to the server $s$ uses $e$.