ETH Zürich
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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2019

Exercise Set 4

> These exercises are **non-graded**. You can submit your solutions at the beginning of next lecture (October 4) or by email (same deadline, to paolo.penna@inf.ethz.ch) in order to get feedbacks.

Exercise 1: (2 + 2 + 2 Points)

Consider the following cost-minimization game, played between two drivers approaching a crossing. The drivers can either stop (S) or cross immediately (C). If they both cross immediately, then they build an accident. This has, of course, high cost for both. If one crosses and one stops, then the player which stops has a small cost for waiting. If both stop, then both have to wait and suffer a small cost.

	C(ross)	S(top)
C(ross)	100 100	1 0
S(top)	0 1	1 1

The numbers in the table are **costs** for the players.

(a) List all pure and mixed Nash equilibria.

(b) Give a correlated equilibrium that is not a mixed Nash equilibrium.

(c) Argue that in this game the set of coarse correlated equilibria coincides with the set of correlated equilibria.

Exercise 2: (2 Points)

The multiplicative-weights algorithm presented in the lecture was stated such that the overall length of the sequence T is given as a fixed parameter. Give a no-external regret algorithm that works without such a parameter for all possible T.

Hint: Use the algorithm from class as a subroutine (you do not need to analyze it again). Start with $T = 1$ as a guess and run the subroutine. Once the subroutine ends, restart it but double your guess.

Exercise 3: (4 Points)

Consider the following algorithm for minimizing external regret (the possible actions are $\{1, 2, \ldots, m\}$ and the costs of each action at time t is always 0 or 1):

Greedy Algorithm (GR)

- Initially set $p^1(a) = 1$ for $a = 1$ and $p^1(a) = 0$ otherwise.
- For each time step $t = 1, \ldots, T$:
  - Let $C^t_{BEST} = \min_a C^t(a)$ and let $S^t = \{a \mid C^t(a) = C^t_{BEST}\}$, where
$$C^t(a) = c^1(a) + c^2(a) + \cdots + c^t(a)\ .$$
  - Set $p^{t+1}(a) = 1$ for $a = \min S^t$ and $p^{t+1}(a) = 0$ otherwise.
    (Choose the minimum element in subset S^t of actions.)

Show that the expected cost achieved by this algorithm on any sequence of T steps is bounded as follows:
$$C^T_{GR} \leq m \cdot C^T_{BEST} + (m - 1)$$
where m is the number of possible actions, and $C^T_{GR} = \sum_{t=1}^T \sum_a p^t(a)c^t(a)$ is the expected of the algorithm.