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Deadline: Beginning of next lecture

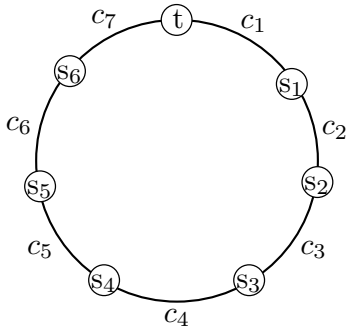
Algorithmic Game Theory

Fall 2019

Exercise Set 5

Exercise 1: (3 Points)

Consider the following fair cost-sharing game:



Each player i wants to be connected from s_i to t, and the edges are **undirected**. That is, two or more players can use the same edge e in different directions, and they all share equally the cost c_e of this edge.

Prove that the price of stability for pure Nash equilibria is at most 2 in this game. (In fact, you can consider any ring with n players.)

Exercise 2: (2 Points)

Consider the following mechanism (A, P) for the shortest path problem (see lecture notes for the setting):

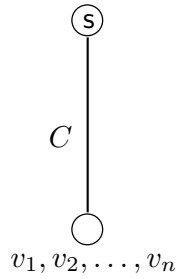
(a) Algorithm A computes the shortest path according to the reported costs $c = (c_1, \ldots, c_n)$.

(b) Each selected player i (i.e., a player whose edge is in the selected path) receives a payment P_i equal to the cost of the alternative shortest path, that is, the length of the shortest path other than the path $A(c)$ computed in the previous step.

Show that this mechanism is **not truthful** on general graphs.

Exercise 3: (3 Points)

Consider the following problem: There are n users (players) potentially interested in receiving a TV transmission, and the transmission is sent from a server s over this simple network:



All users are located in the same node, and $C > 0$ is the **cost** for sending the transmission to one or more users:

> *The server* `s` *can select which users receive the transmission (and which do not). If one or more receive,* `s` *has cost exactly* C *in total (the use of the link). If no user receives, the cost for* `s` *is* 0.

Each user has a **private valuation** v_i, that is, how much he/she is willing to pay for the transmission. Users can chet and report a different valuation r_i. The **utility** of user i is given by the difference between the valuation and the amount of money he/she has to pay:

$$-P_i + \begin{cases} v_i & \text{if } i \text{ receives the transmission} \\ 0 & \text{otherwise} \end{cases}$$

Give a truthful mechanism satisfying the following two conditions:

1. Either only the user with highest (reported) valuation receives and none else does, or none receives the transmission.

2. If one user receives, then he/she is charged an amount which is at least the cost C.

Truthful means that reporting the true valuation v_i maximizes i's utility (no matter how we fix the reports r_j of the other users).