ETH Zürich
Luca Corinzia, Gabriela Malenova,
Ivan Ovinnikov, Paolo Penna

November 1, 2019

Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2019

Exercise Set 7

Exercise 1: (2+2 Points)

Recall the definition of algorithm Greedy-by-Value-Density for combinatorial auction (with single minded bidders) in the lecture notes.

Your task:

1. We have proven that this algorithm is $O(\sqrt{m})$ approximate. Show that **this bound is tight**, that is, the approximation ratio of Greedy-by-Value-Density is at least $\Omega(\sqrt{m})$.

2. We have used the monotonicity of this algorithm to show that there exists payments resulting on a truthful mechanism (payments based on threshold). Show that it is **not possible to use VCG payments** in combination with Greedy-by-Value-Density, and obtain a truthful mechanism in this way. Recall that VCG payments are defined as

$$P_i(b) = Q_i(b_{-i}) - \sum_{j \neq i} b_j(A(b))$$

where $b = (b_1, \ldots, b_i, \ldots, b_n)$ are the input bids, Q_i is an arbitrary function independent of b_i, and A is the algorithm used by the mechanism (here Greedy-by-Value-Density).

Exercise 2: (3 Points)

Consider the following algorithm for single minded combinatorial auction:

Reverse Greedy Algorithm $revGR_\sigma$

1. Start with the set $S^0 \leftarrow \mathcal{N}$ of all bidders;
2. If S^t is not feasible then
   (a) Score bidders in S^t according to some scoring functions[a]
   $$\sigma_i(b_i, b_{\mathcal{N} \setminus S^t}) \tag{1}$$
   where each σ_i is monotone non-decreasing in b_i.
   (b) Drop the lowest score bidder in S^t (break ties arbitrarily).
3. Repeat Step 2 until getting a feasible set S^F (final set of winners).

[a] Here $b_{\mathcal{N} \setminus S^t}$ are the bids of bidders not in S^t, since $\mathcal{N}$ denotes all bidders.

Show that this algorithm can be used in a truthful mechanism, i.e., for every σ as above there exist payment P such that $(revGR_\sigma, P)$ is truthful.

Exercise 3: (2 Points)
Give a direct (self contained) proof that for single minded combinatorial auctions, truthfulness is equivalent to monotonicity of the allocation algorithm.

Definition (monotone allocation rule) For an allocation rule (algorithm) for the single-minded CA problem denote by $W(b)$ the set of winners when the bids are b. We say that this allocation rule is *monotone* if for every i, anf for every b_{-i}, there exists b_i^* such that

$$b_i < b_i^* \Rightarrow i \notin W(b_i, b_{-i}) \tag{2}$$
$$b_i > b_i^* \Rightarrow i \in W(b_i, b_{-i}) \tag{3}$$

Your task: Show that (A, P) truthful implies A monotone and, conversely, if A is monotone then there exist P so that (A, P) is truthful.