ETH Zürich
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Deadline: Beginning of next lecture

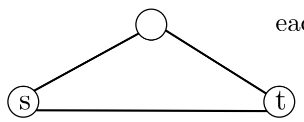
Algorithmic Game Theory

Fall 2019

Exercise Set 8

Exercise 1: (2 Points)

Consider the following scenario (a variant of the in-class exercise of this week – lecture 8):



and we want to sent T **units of traffic** from s to t. Moreover:

1. Each player i has a **private** working **capacity** K_i:

    If i gets more than K_i units of work, each extra unit costs him/her some amount Δ. All the work below K_i has no cost.

2. We give a **fixed compensation** per unit of traffic:

    $$F \cdot w_i$$

    is the payment to player i when he/she gets w_i units of traffic.

Question 1: Model this game as a **single-peaked preferences** when $F < \Delta$.

Question 2: Which outcomes are selected by the **median voter**?

Exercise 2: (2+2+2 Points)

We have three voters and three alternatives X, Y, Z. Consider the following two preference profiles:

P	$\prec_1$	$\prec_2$	$\prec_3$
	X	Y	Z
	Y	X	Y
	Z	Z	X

Q	$\prec'_1$	$\prec'_2$	$\prec'_3$
	Z	Z	X
	Y	X	Y
	X	Y	Z

(1)

Question 1: Show that, if only these two profiles are possible, then *every* social welfare function satisfies **independence of irrelevant alternatives (IIA)**.

Question 2: Suppose that for every player **we know his/her 2^{nd} preference**. Does every social welfare function satisfy independence of irrelevant alternatives? What if we had four alternatives X, Y, W, Z, and we knew for each voter his/her 2^{nd} and 3^{rd} choice?

Question 3: Suppose the possible preferences are all combinations of the individual ranks in (1). That is, all possible profiles are of the form

$$R = (R_1, R_2, R_3) \qquad \text{where } R_i \in \{\prec_i, \prec'_i\} \qquad (2)$$

I propose you the following social welfare function:

1. If voter 1 and 3 agree ($R_1 = R_3$) then return their preference ($F = R_1$);
2. Else ($R_1 \neq R_3$) return some order to be specified ($F = ?$)

Can you have IIA + unanimity, but no dictator?

Exercise 3: (3 Points)

Consider the following facility location problem. We have N feasible locations on the line correspoding to the points $\{1, 2, \ldots, N\}$. There are n players having an ideal (private) position p_i where they would like the facility to be opened, and their cost if facility x is chosen is the distance to the facility $c_i(x) = |x - p_i|$.

Question: Give an incentive compatible (truthful) mechanism which guarantees a **2-approximation** for the **maximum** cost

$$maxcost(x, p) = \max_i c_i(x),$$

where $p = (p_1, \ldots, p_n)$ and $c_i()$ is as above. (The solution should have *maxcost* at most twice the optimal one, no agent should benefit from misreporting p_i, and there are no payments.)