ETH Zürich
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Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2019

Exercise Set 10

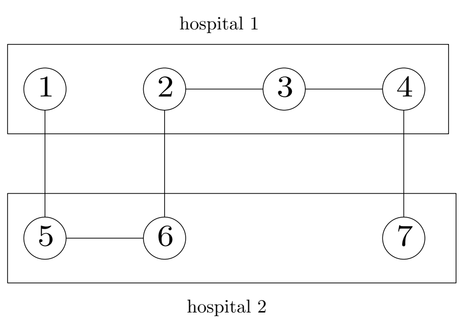
These exercises are **non-graded**. You can submit your solutions at the beginning of next lecture (November 29) or by email (same deadline, to paolo.penna@inf.ethz.ch) in order to get feedbacks.

Exercise 1: (3 Points)

Let us consider **matching mechanisms** applied to the **Kidney exchange** problem involving **two hospitals** (players). Recall the setting:

1. We are given an indirected graph where an edge (u, v) represents mutual compatibility between donor-patient u and donor-patient v (they can be matched).
2. The possible solutions (alternatives) are the matchings over the graph, and the social welfare is the number of matched nodes.
3. Each player i corresponds to a subset of nodes (nodes are partitioned across the players) and the utility of a player is the number of his/her nodes that are matched.

Recall this example from the lecture (see also Roughgarden's book):



Your task: Show that no deterministic truthful mechanism can have an approximation guarantee better than 2 (an α-approximation mechanism returns a matching whose social welfare is at least OPT/α, where OPT is the optimum social welfare; the social welfare of a matching equals the number of nodes that are matched).

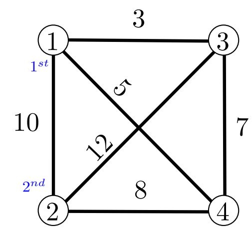
Exercise 2: (2 Points)

In the lecture notes we have shown that TTC Algorithm is truthful for the House Allocation problem. Let us apply the very same algorithm to the Kidney exchange problem, where each player corresponds to a **subset of nodes** and incident edges (an hospital with its donor-patients). Truthfulness now means that no hospital can improve its utility (number of its nodes that get matched) by hiding some nodes to the mechanism.

Is TTCA also truthful for this problem? (Find a counterexample or give a proof.)

Exercise 3: (2 Points)

In **correlated markets** we have a complete weighted graph. Each player (node) prefers neighbors whose edges weights are higher, as shown in this small example:



That is, in a correlated market players have a restricted set of preferences over the other nodes (we still want to match nodes to form a matching). The instances satisfy the following interesting property:

> **Acyclic Instances:** There is no cycle of $\ell \geq 3$ players
>
> $$i_1 \to i_2 \to \cdots \to i_\ell \to i_1$$
>
> such that each player prefers the next one over the previous one.

Prove that indeed any instance of correlated markets (any complete graph) is acyclic.