ETH Zürich
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December 6, 2019

Deadline: Beginning of next lecture

Algorithmic Game Theory

Fall 2019

Exercise Set 12

Your solutions to this exercise sheet will be **graded**. Together with the other three graded exercise sheets, it will account for 30% of your final grade for the course. You are expected to solve the exercises carefully and then write a nice complete exposition of your solution (preferably using **LaTeX** or similar computer editors – the appearance of your solution will also be part of the grade). You are welcome to discuss the tasks with your fellow students, but we expect each of you to hand in **your own** individual write-up. Your write-up should list all collaborators. The deadline for handing in the solution is **Dec 16, before 23:59 (note longer deadline)**. Please send your PDF via email to paolo.penna@inf.ethz.ch. Your file should have the name <Surname>.pdf, where <Surname> should be exchanged with your family name.

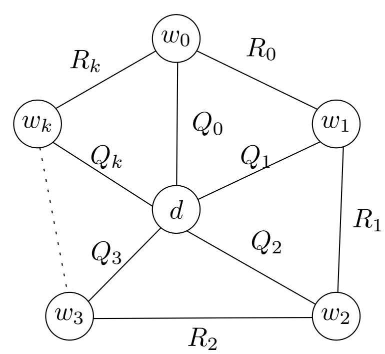
Exercise 1: (3+1 Points)

In this exercise we want to show the implication

$$\text{Gao-Rexford} \Rightarrow \text{No Dispute Wheel} \tag{1}$$

(See lecture notes on BGP for the definitions.)

Consider this kind of simpler wheels (paths R_i and Q_i consist of a single link):



where the preferences of the nodes are

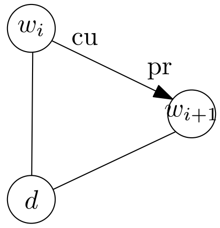
$$Q_i \prec_{w_i} R_i Q_{i+1} \tag{2}$$

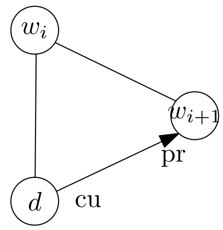
Your task is:

1. Prove (1) for the simple wheels as above.

2. Discuss how to extend the proof to a general wheel.

Hint: Recall that $\emptyset$ denotes any path that does not allow w_i to reach d (in particular if w_{i+1} does not allow transit traffic from w_i) and the utility is $u_{w_i}(\emptyset) = 0$ (the lowest possible). Show that (2) is possible only in one of these two cases:

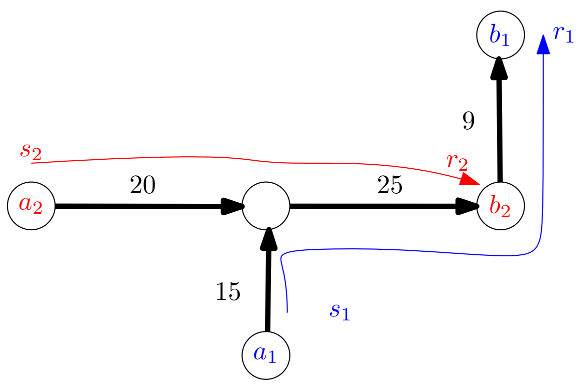




Exercise 2: (4 Points)
Consider TCP games on **general networks** where each edge is a channel of some capacity. Each player sends at a certain rate s_i along a predetermined path, and his/her utility is the rate r_i at which the traffic arrives at the destination (see below for an example).

Show that, if all channels use the same **Strict Priority Queuing** policy (see lecture notes), then the result proved for a single channel extends (the game is NBR-solvable with clear outcome and therefore PIED converges and is incentive compatible).

Here is an example:



In this example, if both players send at rate $s_1 = s_2 = 20$, their respective utility is $r_1 = 9$ and $r_2 = 10$. Indeed, the channel of capacity 15 will allow a rate of 15 from player 1. Therefore, the next edge in the path, the channel of capacity 25, will receive a flow of 15 from player 1 and 20 from player 2. Because of Strict Priority, this channel assigns 15 to player 1 and $25 - 15 = 10$ to player 2. The last edge in the path of player 1 will assign 9 to it, which gives his/her rate r_1.

Note: We assume that the edges are **directed**, and each edge e is a channel of capacity C_e. For any player i, we have a corresponding source-destination pair (a_i, b_i) connected by a fixed path π_i. Player i has a maximum transmission rate M_i, and thus the strategy is the sending rate $s_i \in [0, M_i]$. Each channel e divides its capacity among the players whose path contains this edge as follows. If an edge e in the path of i assigns some rate $r_{i,e}$ to this player, then **the next edge** e' in this path receives a rate $r_{i,e}$. Channel e' applies the Strict Priority to all received rates and determines $r_{i,e'}$. The received rate $r_i(s)$ is the rate that the destination b_i gets from a_i (i.e., the rate that the last edge e in π_i assigns to i).

Exercise 3: (3+1 Points)

In this exercise we want to show that the VCG mechanism for sponsored search satisfies the following two conditions:

- **Envy-freeness** meaning that no bidder getting slot s would like to get slot $s+1$ and pay the price of bidder $s+1$, nor slot $s-1$ and pay the price of bidder $s-1$:

$$\alpha_s v_s - P_s^{VCG}(v) \geq \alpha_t v_s - P_t^{VCG}(v) \qquad \text{for } t \in \{s-1, s+1\} \tag{3}$$

- **Voluntary partecipation** which is the usual condition that truth-telling bidders have non-negative utilities.

Prove that both conditions indeed hold.