Graphs and Algorithms

Hypercube

The $d$-dimensional hypercube $Q_d$ is a graph with vertex set $\{0, 1\}^d$. There is an edge between two vertices $u, v \in \{0, 1\}^d$ if and only if $u$ and $v$ differ in exactly one coordinate. Show that $Q_d$ has a hamilton cycle for $d \geq 2$.

TSP Approximation Algorithms

a) In the lecture you have seen a 2-approximation algorithm (based on the construction of an MST) for the metric TSP problem. The goal of this exercise is to improve the approximation ratio to $3/2$.

**Hint:** Extend the MST such that it contains an Euler tour. You can assume that it is possible to compute a minimum weighted perfect matching in polynomial time.

b) Show that for every $\alpha \geq 1$, it is not possible to obtain an $\alpha$-approximation for the general (non-metric) TSP problem in polynomial time unless $P = NP$.

Discussion of the exercises on 03.05.2007.