Graphs and Algorithms

Greedy Coloring

a) Order the vertices of a graph $G = (V, E)$ according to their degrees, so that $V = \{v_1, v_2, \ldots, v_n\}$ and $d(x_1) \geq d(x_2) \geq \cdots \geq d(x_n)$. Show that in this order the greedy coloring algorithm uses at most $\max_i \min\{d(x_i) + 1, i\}$ colors, and so if $k$ is the maximal natural number for which $k \leq d(x_k) + 1$ then $\chi(G) \leq k$.

b) Deduce from this that if $G$ has $n$ vertices then

$$\chi(G) + \chi(\overline{G}) \leq n + 1$$

where $\overline{G}$ is the complement of $G$, i.e., $V(G) = V(\overline{G})$ and for all $u, v \in V(G)$, $\{u, v\} \in E(\overline{G})$ iff $\{u, v\} \notin E(G)$.

c) Show that $\chi(G) + \chi(\overline{G}) \geq 2\sqrt{n}$.

d) Show that for both bounds there are tight examples for infinitely many $n$.

Discussion of the exercises on 31.05.2007.