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Programming Exam

Algorithmen und Datenstrukturen

January 27, 2020

DO NOT OPEN!

Last name, first name: ______________________________

Student number: ______________________________

With my signature I confirm that I can participate in the exam under regular conditions. I will act honestly during the exam, and I will not use any forbidden means.

Signature: ______________________________

Good luck!

The **ENROLLMENT PASSWORD** is “`mergesort`”.

1 Relations and Directed Graphs, 24 points

You are given a directed graph $G = (V, E)$ with $V = \{0, \ldots, n-1\}$ represented by an adjacency matrix `diGraph`, and your task is to implement 3 methods on it.
For two of these methods, the edge set E will represent a relation on V. Recall that a relation on R is given by a set $R \subseteq V \times V$, and we set $R := E$. In other words, $(a, b) \in R$ if and only if $(a, b) \in E$.

As a reminder, here are two definitions from Diskrete Mathematik (but note that this exercise is self-contained):

- The relation R is **antisymmetric**, when

$$\forall a, b \in V : \Big((a, b) \in R \wedge (b, a) \in R \implies a = b\Big).$$

- The relation R is **transitive**, when

$$\forall a, b, c \in V : \Big((a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R\Big).$$

You should implement the following three methods (see next page for examples):

a) `isAntisymmetricRelation()` tests if the relation $R = E$ is antisymmetric.

   You can get **4 points** with an algorithm of **runtime $\mathbf{O(|V|^2)}$**.

b) `isTransitiveRelation()` tests if the relation $R = E$ is transitive.

   You can get **4 points** for **runtime $\mathbf{O(|V|^3)}$**

c) `containsStar` tests if the graph contains a *star*. A star is a vertex which has indegree $|V| - 1$ and outdegree 0 and no self-loop.

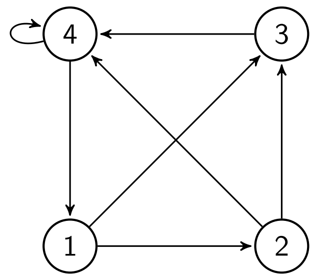
   The input is a sorted array of edges, and as such, it is in a separate class. The sorting is primarily according to the source vertex, secondarily according to the target vertex. E.g., the edge $(1, 5)$ comes before $(2, 3)$, and that comes before $(2, 4)$.

   You can get **2 points** with an algorithm of **runtime $\mathbf{O(|V| \cdot |E|)}$** and **6 points** (in total) for **runtime $\mathbf{O(|E|)}$**.

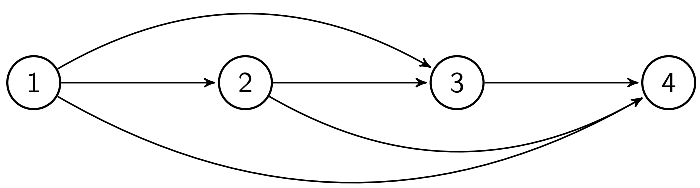
d) `containsCycle()` tests if the graph contains a directed cycle.

   You can achieve up to **10 points** with a correct algorithm of **runtime $\mathbf{O(|V|^2)}$**. You can get partial points for a set of particularly easy instances (**2 points**), or for **runtime $\mathbf{O(|V|^3)}$** (**2 points**). The remaining **6 points** are for the generic case.

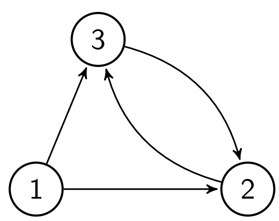
Examples:



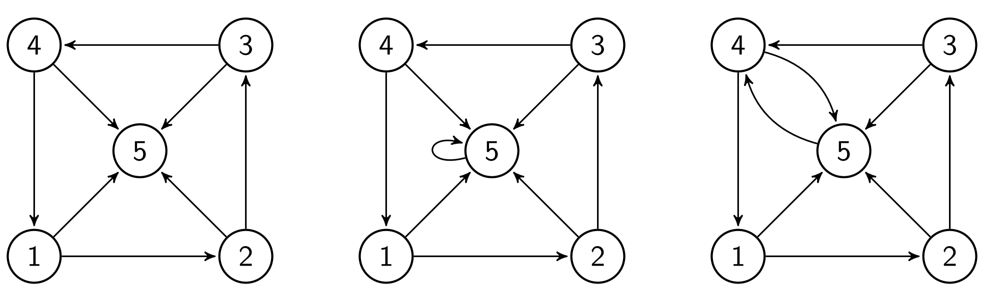
- `isAntisymmetricRelation()` returns `true`.
- `isTransitiveRelation()` returns `false`.
- `containsCycle()` returns `true`.



- `isAntisymmetricRelation()` returns `true`.
- `isTransitiveRelation()` returns `true`.
- `containsCycle()` returns `false`.



- `isAntisymmetricRelation()` returns `false`.
- `isTransitiveRelation()` returns `false`.
- `containsCycle()` returns `true`.



- `containsStar()` returns (from left to right) `true`, `false` (star cannot have self-loop), and `false` (star cannot have outgoing edge).

2 String splitter, 16 points

You are given a string s that contains only characters `a`, `b`, ..., `z` and no spaces. You are also given a dictionary D of words using the same characters. Your task is to find the length of the longest initial sequence s^* of s (i.e., starting from position 0) that can perfectly be split into words from the dictionary. I.e., s^* can be partitioned into consecutive substrings that are all in D.

Words from the dictionary may be used more than once.

Several **Examples:**

- For string $s =$ `aabb` and dictionary { `b`, `c` } the output is 0. (You have to start from the beginning and `a` is not in the dictionary).
- For string $s =$ `aabb` and dictionary { `a`, `c` } the output is 2. (The initial string $s^* =$ `aa` can be split in the form `a|a`, where `|` marks split).
- For string $s =$ `aabb` and dictionary { `a`, `b` } the output is 4. (The initial string $s^* = s =$ `aabb` can be split in the form `a|a|b|b`).
- For string $s =$ `aabb` and dictionary { `a`, `abb` } the output is 4. (The initial string $s^* = s =$ `aabb` can be split in the form `a|abb`).

We use the following notation.

- $|s|$ is the length of the string s.
- $|D|$ is the total number of characters in the dictionary.

The tests contains 4 **test sets**:

- `unique`: the input dictionary is composed of words with unique last character. **2 points**.
- `small`: the general case, but the time limit is generous. **3 points**.
- `large`: the general case, but your solution has to be efficient. A runtime of $\mathcal{O}(|s| \cdot |D|)$ is fast enough, but other efficient implementation with different asymptotic runtimes may also be accepted. **9 points**.
- `extra_large`: a challenge. The input is the same as "large", but an asymptotically even faster runtime is required. **2 points**.

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