Randomized Algorithms and Probabilistic Methods:
Advanced Topics

Exercise 1
Let $Q_1 \subseteq nB$ and let $Q_2 \subseteq n^2B \setminus nB$ with $|Q_1| + |Q_2| = \text{poly}(n)$. A convex body $K$ is consistent (with $Q_1$ and $Q_2$) if $Q_1 \subseteq K$ and $K \cap Q_2 = \emptyset$. Let $K_1$ be the smallest consistent body and $K_2$ be the largest consistent body with $K_2 \subseteq n^2B$. Show

$$\frac{2^n}{\text{poly}(n)} \cdot \text{vol}(K_1) \leq \text{vol}(K_2).$$

Hint: Show that the convex hull $CH$ of $Q_1$ is a subset of $\bigcup_{p \in CH} B_2(p/2, ||p/2||)$. In order to do that, show that if a point $x$ is not in $B_2(p/2, ||p/2||)$ for some $p \in CH$, then $p$ is in the open halfspace which is bounded by the hyperplane through $x$ perpendicular to $x$ and contains the origin.

Exercise 2
Let $K$ be a convex body in $\mathbb{R}^n$ such that $B \subseteq K \subseteq 2^{\text{poly}(n)}B$, where $B$ is the infinity norm ball with radius 1. Show that there exists an affine transformation $f$ such that $B \subseteq f(K) \subseteq n^2B$ and that $f$ can be computed in $\text{poly}(n)$ time. You may assume that you can optimize a linear function over $K$ in $\text{poly}(n)$ time.