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2nd March 2016

Datenstrukturen & Algorithmen — Exercise Sheet 2 — FS 16

Exercise 2.1 *Estimating Asymptotic Running Time.*

Specify (as concisely as possible) the asymptotic running time of the following Java code fragments in Θ notation depending on $n \in \mathbb{N}$.

```
for (int i = 1; i <= n; i = i · 2) {
        for (int j = n; j > 1; j −= 10)
                ;
}
```

```
for (int i = n; i > 0; i −= 5) {
        for (int j = 0; j < i; j += 1) {
                int k = 1;
                while (k · k <= n)
                        k = k + 2;
        }
}
```

```
int f (int n) {
        if (n == 1) return 1;
        else {
                for (int i = 1; i <= 2n; i ++)
                        ;
                return f(n/2)+1;
        }
}
```

Exercise 2.2 *Recurrence Relations.*

Given a recurrence relation of the form

$$T(n) = \begin{cases} aT(\frac{n}{b}) + cn + d & \text{if } n > 1 \\ e & \text{if } n = 1 \end{cases}$$

with $a, b, c, d, e \in \mathbb{N}$, $a \neq b, a \neq 1$ and $b > 1$. Find an expression with summations by telescoping and use the summation formula for geometric series ($\sum_{i=0}^{k} q^i = \frac{q^{k+1}-1}{q-1}$ for $q \neq 1$) to find a closed form. Prove your answer using mathematical induction. You can assume that n is a power of b.

Please turn over.

Exercise 2.3 *Open Hashing.*

a) Insert the keys $10, 18, 2, 4, 17$ in this order into an initially empty hash table of size 7. Use open addressing with the hash function $h(k) = k \bmod 7$ and resolve the conflicts using

   (i) linear probing (to the left)

   (ii) quadratic probing (to the right, to the left, to the right, ...), and

   (iii) double hashing with $h'(k) = 1 + (k \bmod 5)$.

   For each method, provide the number of collisions. Which one is best for the above situation?

b) Which problem occurs if the key 10 is removed from the hash tables in a), and how can you resolve it? Which problems occur if many keys are removed from a hash table?

c) Provide a sequence of insert operations such that quadratic probing causes more collisions than linear probing. Specify a rule that prescribes how to form such a sequence of *arbitrary* length n. Use the hash function $h(k) = k \bmod m$ for a prime number m with $m \geq n$.

Exercise 2.4 *Cuckoo hashing.*

Cuckoo hashing is a hashing technique that guarantees constant time *in worst case* for both query and delete operations. The idea is to use two tables T_1 and T_2 of same size and two hash functions h_1 and h_2. A new key x is inserted at position $h_1(x)$ in T_1. In case of a collision, the previously stored key y is displaced to position $h_2(y)$ in T_2. If this leads to another collision, the next key is again inserted at the appropriate position in T_1.

In some cases, this procedure continues forever, i.e. the same configuration appears after some steps of key displacements due to collisions. We will illustrate such a case in this exercise.

a) Given two tables of size 5 each and two hash functions $h_1 = k \mod 5$ and $h_2 = \lfloor k/5 \rfloor \mod 5$. Insert the following keys in the initially empty hash tables in this order: $3, 16, 18, 23, 1$.

b) Find another key, such that the insertion leads to an infinite sequence of key displacements. (In such a case, one would define two new hash functions, allocate two new tables and store the keys in these tables using the new hash functions.)

Hand-in: Wednesday, 9th March 2016 in your exercise group.