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Datenstrukturen & Algorithmen — Exercise Sheet 4 — FS 16

Exercise 4.1 *Search Trees.*

a) Draw the resulting tree if you insert the keys $4, 8, 16, 1, 7, 12, 6, 5$ in this order into an initially empty natural search tree.

b) Give the preorder, postorder, and inorder traversal of the tree in a).

c) Remove the key 1 from the tree in a) and after that the key 8 from the resulting tree. Draw both trees.

d) Draw the result if the keys from part a) are inserted into an initially empty AVL tree.

Exercise 4.2 *Sorting algorithms.*

Answer the following questions and give a brief explanation of your answer.

a) Is the sorted sequence $1, 2, \ldots, n$ a Min-Heap?

b) When all the elements in a Min-Heap are different, at which positions could the largest element be found?

c) A comparison-based algorithm is called *stable* if the relative order of identical elements is not changed. A sorting algorithm is called *in-situ* if it works on the input sequence using only a constant amount of additional space for storing parts of the sequence. Which comparison-based sorting algorithms that you know are stable and which are in-situ, or can easily be adapted accordingly?

Please turn over.

Exercise 4.3 *Two-dimensional Maximum Subarray Problem(**Programming Exercise**).*

For a given $(n \times n)$ integer matrix $A = (a_{ij})_{1 \le i,j \le n}$, the goal is to compute a submatrix with the largest sum of entries. A $(a \times b)$ submatrix of a $(n \times n)$ matrix arises by considering a continuous $(a \times b)$ block of the entries of A $(0 \le a, b \le n)$.

Input The first line of the input contains only the number t of testcases. Each of the t testcases is then given in the following way: the first line contains $n \le 100$. After that, we have n lines of n integer numbers representing A. The i-th line corresponds to the i-th row of A, i.e. it contains $a_{ij} \le 100$ for $1 \le j \le n$.

Output Output the largest sum of a submatrix for each testcase on a separate line.

Example

Input:

```
3
2
-1 3
3 -1
2
-2 -3
-1 -4
2
2 -1
-2 -1
```

Output:

```
4
0
2
```

Remarks As you can see from the second testcase above, also the empty (0×0) submatrix is a valid solution. There are three categories of testsets for a total of 100 points:

- **Easy**: For the easy testset we have $n \le 20$. Worths 20 points.
- **Medium**: For the medium testset we have $n \le 50$. Worths 30 points.
- **Hard**: For the hard testset we have $n \le 100$. Worths 50 points.

Hints: The fact that we have three different testsets makes it possible to differentiate three different time complexities. The easy testset can be solved by simple brute-force, which takes $O(n^6)$ time. The medium testset requires a simple precalculation which can improve the runtime to $O(n^4)$. For the hard testset you have to reduce to the maximum subarray problem that we saw in the course. This will lead to an $O(n^3)$ algorithm. Of course, if your algorithm solves a testset on time, then it solves all the easier ones too.

Hand-in: Wednesday, 23rd March 2016 in your exercise group.