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Datenstrukturen & Algorithmen Exercise Sheet 13 FS 16

Exercise 13.1 *Matchings.*

Provide a connected graph with 6 vertices that contains **exactly** 3 different perfect matchings. A matching is *perfect* if every vertex is matched to exactly one other vertex.

The following two exercises deal with scanline algorithms for solving geometrical problems. Every description of a scanline algorithm should address the following aspects.

a) Stopping points. *In which direction is the scanline moving? What are the stopping points?*

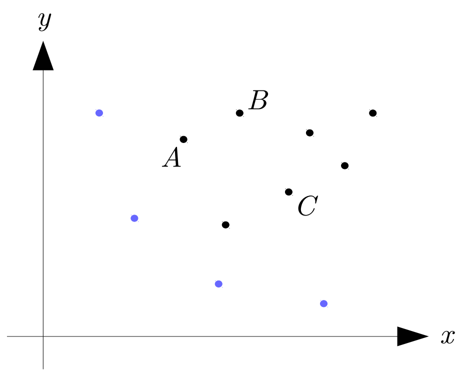
b) Scanline data structure. *Which objects have to be stored in the data structure? Which operations have to be supported? What is an appropriate data structure?*

c) Update. *What happens if the scanline encounters a new stopping point?*

d) Extracting the solution. *How can the solution be extracted?*

Exercise 13.2 *Non-Dominated Points.*

You are given a set of points in the plane. A point (x, y) *dominates* some other point (x', y') if we simultaneously have both $x < x'$ and $y < y'$. For example, in the image on the right the point B is dominated by the point A, but not by the point C. The set of all non-dominated points is drawn in blue.

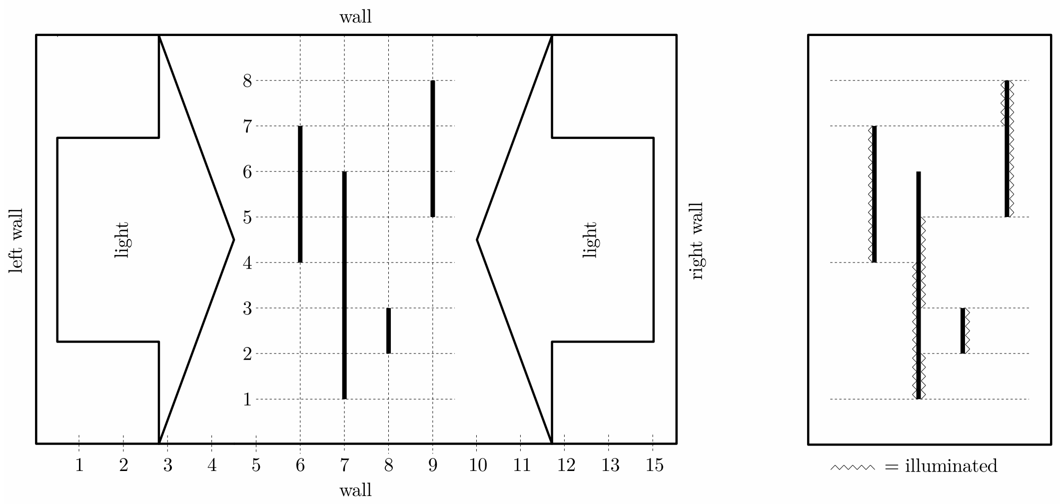


Design a scanline algorithm that, given a set of n points, computes the set of all non-dominated points as efficient as possible. Provide also the running time of your solution.

Exercise 13.3 *Artwork* ***(Exercise in the exam in January 2015)****.*

An artist draws a ground plot of his artwork consisting of plates that are illuminated with lasers from both sides.

There are n plates with different positions and widths. The i-th plate is represented by a triple $P_i = (x_i, y_i, b_i)$, where x_i is the distance to the left wall, y_i is the distance to the lower wall in the picture above, and b_i is the width of the plate. For example, a possible input for the following ground plot is $P_1 = (7, 1, 5)$, $P_2 = (9, 5, 3)$, $P_3 = (8, 2, 1)$, and $P_4 = (6, 4, 3)$.



The plates are illuminated from extensive laser light sources at the left and the right wall. The plates absorb all incident light. As the plates are heated by the light, they need to be cooled proportionally to the amount of incident light. Therefore, the artist wants to compute the illuminated area (i.e. the sum of the area illuminated by the light source on the left wall and the light source on the right wall) *of each plate*. Instead of computing the illuminated area it is sufficient to compute the illuminated width, because all plates have the same height. Therefore, it is sufficient to consider the two-dimensional problem as shown in the above sketch. For the sake of simplicity we assume that no two plates are positioned directly on the top of each other and that no two plates start and end directly next to each other. Note that (as in the above example) the plates are not necessarily ordered by their x-coordinates.

For each plate i the total illuminated width B_i, i.e. the sum of the width illuminated from the left and the width illuminated from the right, should be computed (cf. the right sketch above). The result for the above instance is $B_1 = 6$, $B_2 = 4$, $B_3 = 1$, and $B_4 = 3$.

Design an efficient sweepline algorithm for the above problem. Provide the running time of your algorithm depending on n, and justify your answer.

Exercise 13.4 *Dating agency (Programming exercise).*

You are working for a dating site company. The users of the site are split in two disjoint sets: Men M and Women W. They answer various questions and then an algorithm is run to decide who is compatible with whom. This algorithm declares a pair of users (m, w), $m \in M$ and $w \in W$, as compatible when they both like each other (which is determined according to their answers).

Every user can only be *matched* to at most one other user. Of course, it is in the company's interest to identify as many matches as possible. Your job is to implement an algorithm that solves the following problem: Given a list of pairs of users (m, w) with $m \in M$ and $w \in W$ that are compatible compute what is the maximum number of possible matched pairs.

Input The first line contains only the number T of test instances. Then for each instance: The first line only contains the numbers $|M|$ and $|W|$. The second line only contains the number of compatible pairs p. Then, each pair is declared on a separate line as m w. This means that users $m \in M$ and $w \in W$ are compatible.

Output For every test instance we output only one line. This line contains the value of the maximum number of matched pairs.

Example

Input:

```
2
4 4
5
1 2
1 3
2 4
3 1
4 1
3 3
4
1 3
2 1
3 1
3 2
```

Output:

```
3
3
```

Directions There is only one testset for 100 points in this exercise.

Hand-in: Wednesday, 1st June 2016 in your exercise group.