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Algorithmic Game Theory HS 2014

Exercise sheet 9

EXERCISE 9.1:

In this exercise we consider combinatorial auctions with single-minded bidders. Recall that in such an auction every player is only interested in getting the goods in $S_i \subseteq U$ (where U is the set of goods). The player i values this bundle S_i with $v_i \in \mathbb{R}^+$. Both S_i and v_i are the private information of player i. Every player i submits a bid (B_i, b_i) to the auction, expressing the desire to get the bundle B_i and that the player values it with b_i.

Recall the characteristics of VCG and the LOS mechanisms. In VCG the mechanism computes an optimal allocation $\{S_i^*\}_{i=1}^n$ of goods to the players (where the allocation maximizes the sum of the valuations of all players), and the payment p_i to every player i:

$$p_i = \sum_{j \neq i} b_j(\bar{S}_j) - \sum_{j \neq i} b_j(S_j^*),$$

where $\{\bar{S}_j\}_{j \neq i}$ is an assignment maximizing the total valuation of players $1, 2, \ldots, i-1, i+1, \ldots, n$.

In a LOS mechanism a greedy algorithm is used to compute an approximate solution. In each iteration it grants the bid with the highest value according to the formula $b_i/\sqrt{|B_i|}$, after which it removes the bids that are blocked by B_i before reiterating. The payment to a player i is then $q_i = b_j\sqrt{|B_i|/|B_j|}$, where player j is the highest uniquely blocked bidder of i. In both mechanisms a player who is not granted his bid pays nothing.

a) Consider the VCG mechanism and the LOS mechanism for a combinatorial auction with single-minded bidders.

   Provide a problem instance for each one of the following settings:

   i) The total sum of payments in the VCG mechanism is greater than the total sum of payments in the LOS mechanism.

   ii) The total sum of payments in the LOS mechanism is greater than the total sum of payments in the VCG mechanism.

b) Consider the LOS greedy algorithm for granting bids of players. In the lecture we have seen that a player i with her bid (B_i, b_i) can uniquely block (u-block for short) a player j with her bid (B_j, b_j) even if $B_i \cap B_j = \emptyset$. Show, however, that if j is the highest u-blocked bid by player i, then $B_i \cap B_j \neq \emptyset$.

c) Consider the following modification of the LOS mechanism:

   i) the outcome (i.e., the decision of the mechanism about which player is granted its bundle) remains unchanged;

   ii) the price that any winner i pays is $\sqrt{|B_i|}\frac{b_j}{\sqrt{|B_j|}}$, where $j > i$ is the first j after i (in the order given by the descending values of $b_k/\sqrt{|B_k|}$, $k = 1, \ldots, n$) for which $B_i \cap B_j \neq \emptyset$. The payment will be zero if no such j exists.

   Is this mechanism truthful?

EXERCISE 9.2:
Recall the problem of scheduling m jobs on n machines, where every job j has a load (size) l_j, and every machine i needs t_i time to process one unit of load. The machines are the players and t_i is the private information (its type) of player i. Every player i submits to the mechanism value b_i with which it claims the time to process a unit of load of machine i to be b_i. The mechanism then assigns to every machine i a set of jobs J_i such that $J_1, J_2, ..., J_n$ forms a partition of the jobs $\{1, 2, \ldots, m\}$, and decides for every player i the amount of money p_i the player i gets. The load (or work) of machine i in this assignment is $W(i) = \sum_{j \in J_i} l_j$. The utility of player i is $u_i = p_i - t_i \cdot W(i)$. The expression $t_i \cdot W(i)$ is the cost to machine i.

Consider the following greedy strategy for assigning jobs to machines: Sort the jobs such that $l_j \geq l_{j+1}$; Go through the jobs in the sorted order, and assign job j to a machine i iteratively (to be specified in the following); Let $W^{(j-1)}(i)$ denote the load of machine i after the first $j - 1$ jobs were assigned; Assign job j to machine i which minimizes the value $b_i \cdot W^{(j-1)}(i) + b_i \cdot l_j$ (i.e., minimizing the time when machine i finishes when job j is assigned to it) where ties are broken arbitrarily. Can you design prices such that this algorithm and the designed prices form a truthful mechanism?