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Algorithmen & Datenstrukturen Exercise Sheet 12 AS 16

Hand-in: Thursday, 15th December 2016 before the start of the lecture at 10:00 in the entrance area of ML D28. Please staple all sheets together and use this sheet as the cover page. Fill out the first two fields of the form below.

Exercise class (Room & Day): ____________________

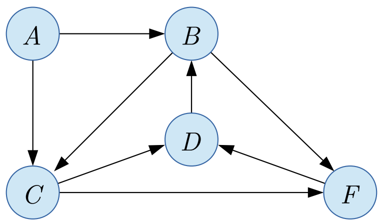
Submitted by: ____________________

Corrected by: ____________________

Bonus points: _____

Exercise 12.1 *Topological sorting and connected components.*

a) Given the following graph $G = (V, E)$, specify a set $E' \subset E$ of smallest possible cardinality, such that $G' = (V, E \setminus E')$ can be sorted topologically. Justify your choice.



b) Provide a topological ordering for the graph G' from a).

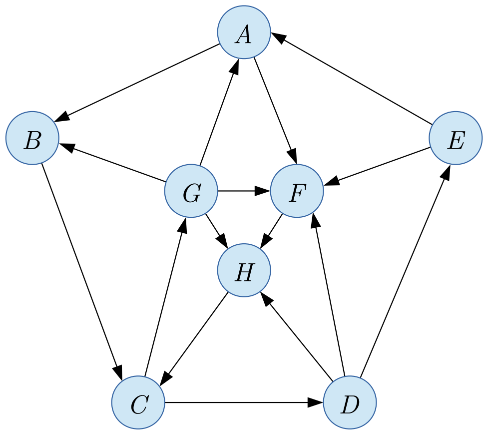
c) For each $n \in \{1, 2, 3, 4\}$, construct a graph with n vertices that can be sorted topologically and has a maximal number of edges. For a general n, how many edges can a graph with n vertices have at most, such that it can be sorted topologically? Prove your answer.

d) How many edges has an undirected graph with n vertices and $k \in \{1, \ldots, n\}$ connected components at least? Describe how connected components with a minimal number of edges look, and justify your answer.

Exercise 12.2 *Depth-first search and breadth-first search.*

Every depth-first search and breadth-first search has some degree of freedom, namely the order, in which the neighbors of a vertex are visited. We assume that the depth-first search, respectively the breadth-first search, visits the neighbors in alphabetically ascending order. We call the order, in which the depth-first search, respectively the breadth-first search, visits the vertices of the graph *depth-first search order*, respectively *breadth-first search order*.

a) Provide the depth-first search order and the breadth-first search order of the graph on the right, starting at vertex A.
b) Is there a vertex in the graph on the right, such that the depth-first search order starting at this vertex is equal to the breadth-first search order starting at this vertex and vice versa? Justify your answer.
c) Given some arbitrary $n \in \mathbb{N}$, provide an *undirected*, connected graph with n vertices, such that the breadth-first search order starting at an arbitrary vertex is also a depth-first search order and vice versa.
d) What is the asymptotic running time of the depth-first search and the breadth-first search when applied on an adjacency matrix instead of an adjacency list? Justify your answer.



Exercise 12.3 *Black Holes.*

Let $G = (V, E)$ be a directed graph. A *black hole* is a vertex $v \in V$ with an indegree of $|V| - 1$ and an outdegree of 0. Describe an algorithm that gets the adjacency matrix of the graph $G = (V, E)$ as input, and that checks whether G contains a black hole or not by considering only $\mathcal{O}(|V|)$ many matrix entries.

Note: Of course, a black hole can be found in time $\Theta(|V|^2)$ by considering all entries of the adjacency matrix. We search for a more efficient solution.